Computer Algebra Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.2-Cosine/84-4.2.1.1-a+b-cos-^n

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September 5, 2023 Compiled on September 5, 2023 at 2:40pm

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [62]. This is test number [84].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (62)	0.00 (0)
Mathematica	100.00 (62)	0.00 (0)
Maple	72.58 (45)	27.42 (17)
Fricas	72.58 (45)	27.42 (17)
Giac	62.90 (39)	37.10 (23)
Maxima	62.90 (39)	37.10 (23)
Mupad	56.45 (35)	43.55 (27)
Sympy	51.61 (32)	48.39 (30)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

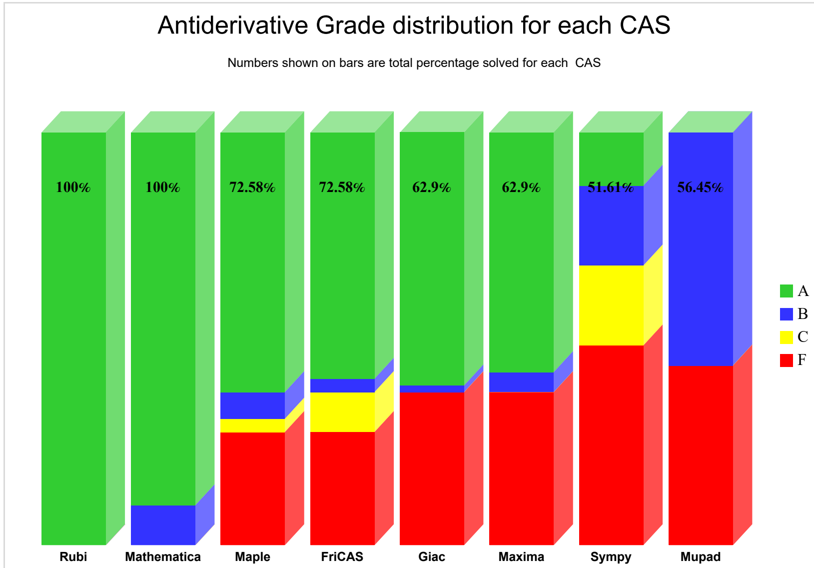
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

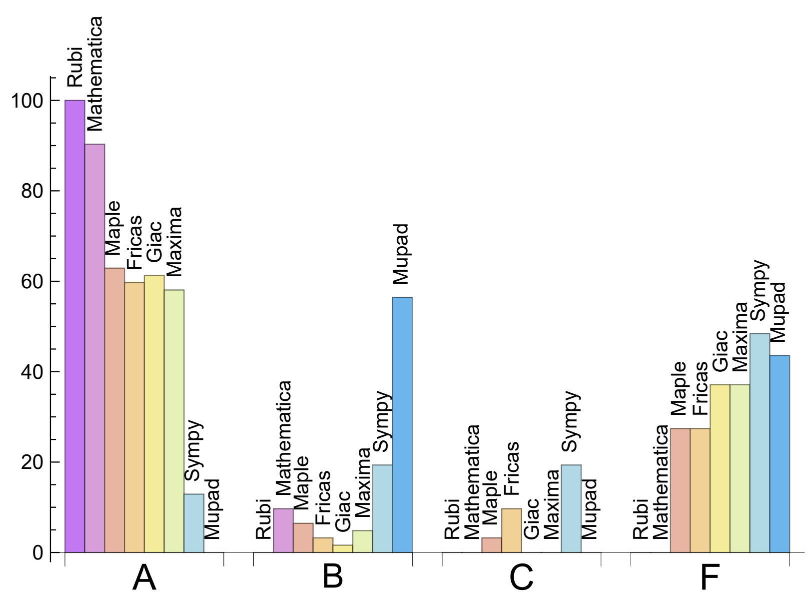
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	90.323	9.677	0.000	0.000
Maple	62.903	6.452	3.226	27.419
Giac	61.290	1.613	0.000	37.097
Fricas	59.677	3.226	9.677	27.419
Maxima	58.065	4.839	0.000	37.097
Sympy	12.903	19.355	19.355	48.387
Mupad	0.000	56.452	0.000	43.548

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	17	100.00	0.00	0.00
Maple	17	100.00	0.00	0.00
Giac	23	100.00	0.00	0.00
Maxima	23	100.00	0.00	0.00
Mupad	27	0.00	100.00	0.00
Sympy	30	96.67	3.33	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.08
Fricas	0.24
Mathematica	0.25
Giac	0.29
Maxima	0.76
Maple	1.11
Sympy	1.30
Mupad	10.54

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	65.26	0.85	67.00	0.90
Giac	70.13	0.88	75.00	0.83
Rubi	88.03	1.00	83.00	1.00
Maple	103.73	1.03	71.00	0.77
Mathematica	104.81	1.12	69.00	1.00
Fricas	135.69	1.41	90.00	1.11
Sympy	346.69	3.51	299.50	3.76
Maxima	2655.69	26.49	94.00	1.19

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

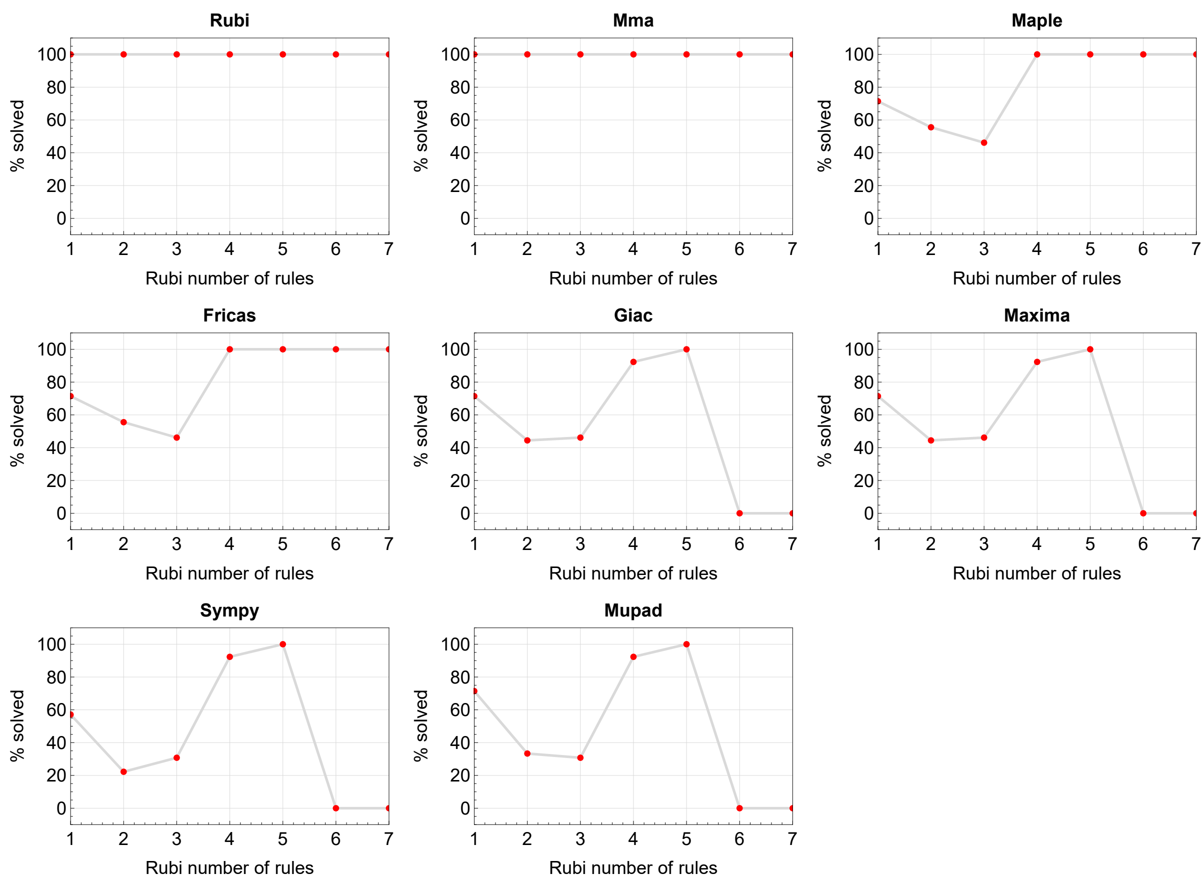


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

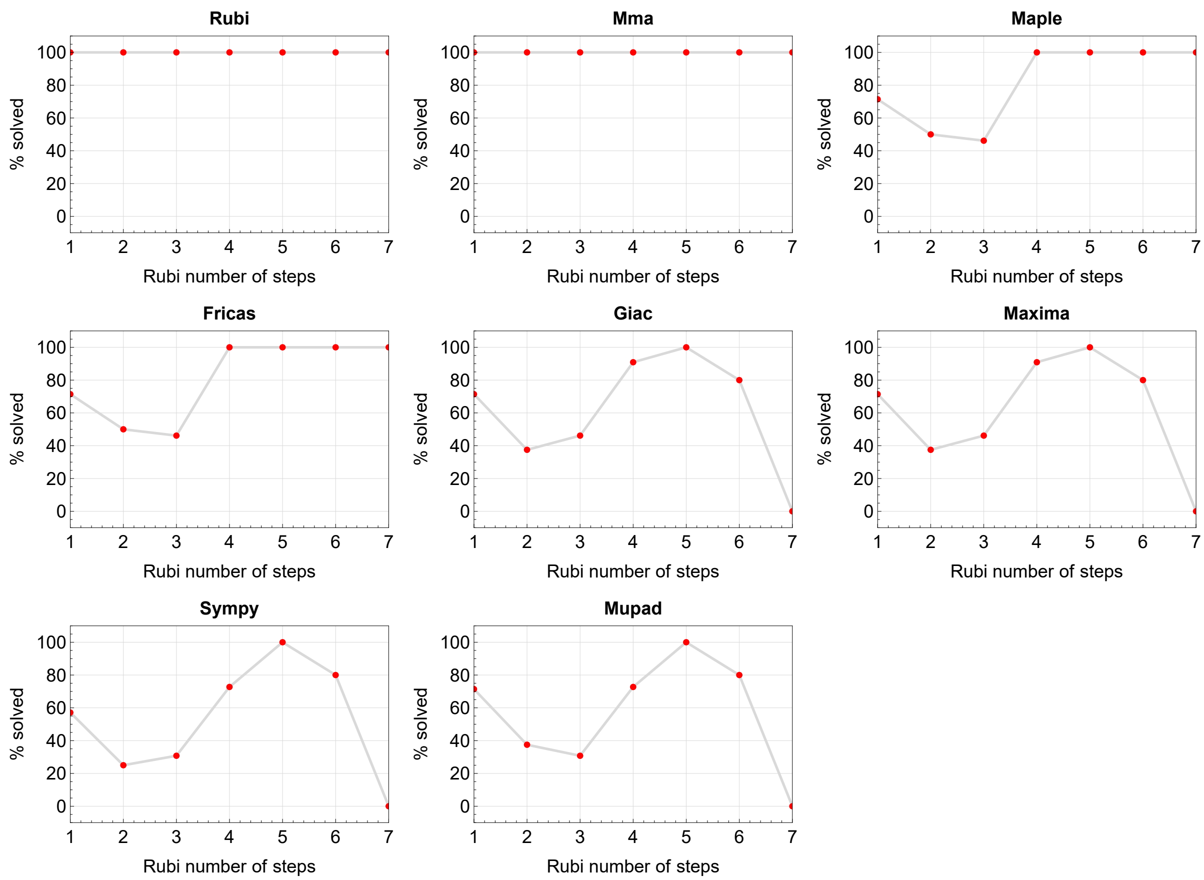


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

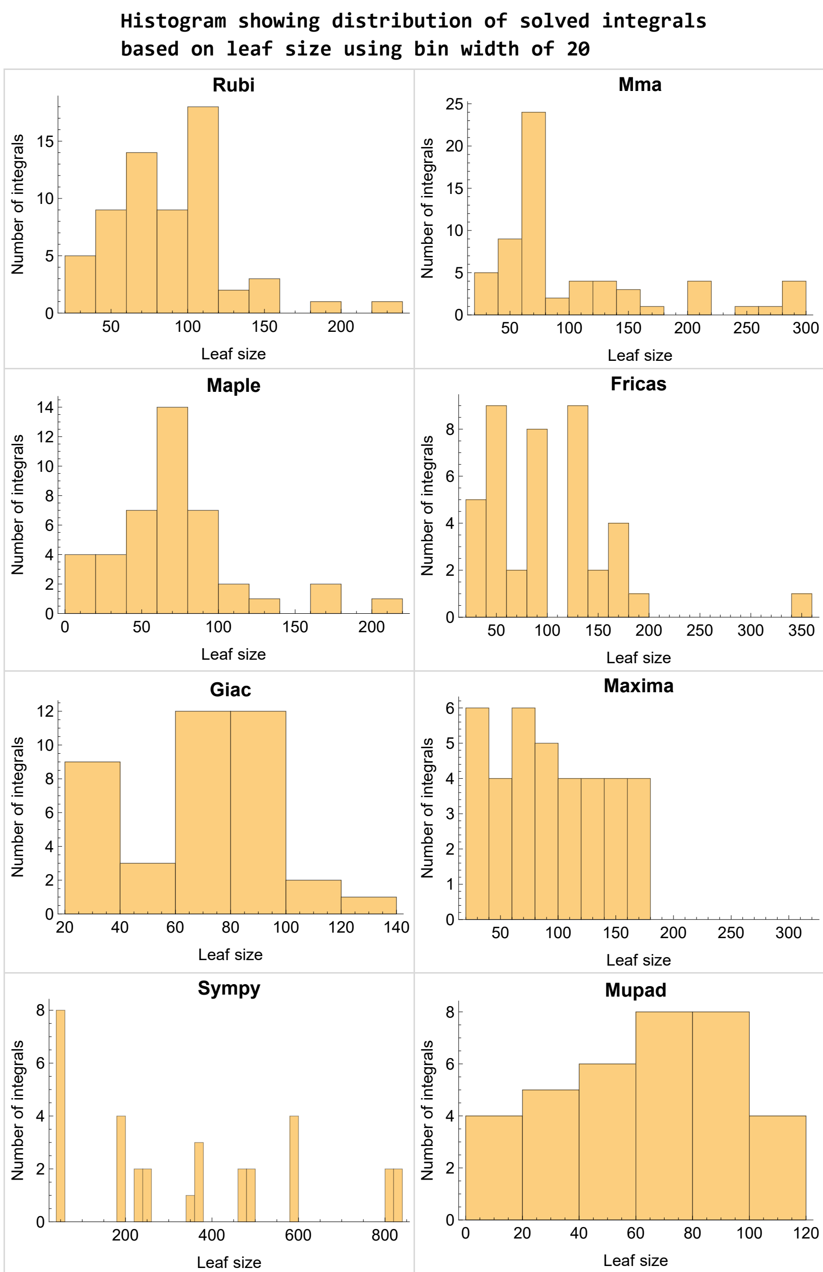


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

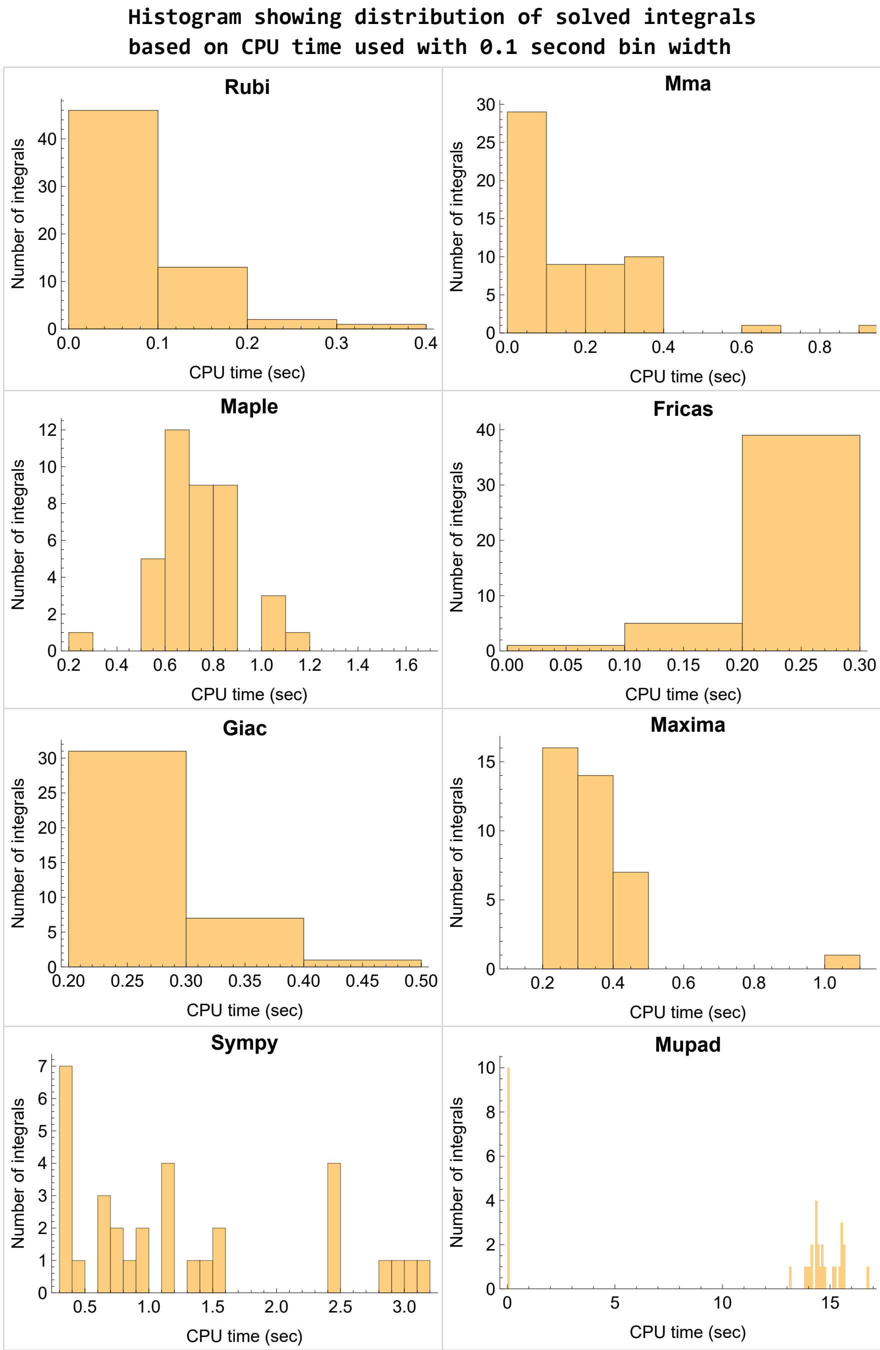


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

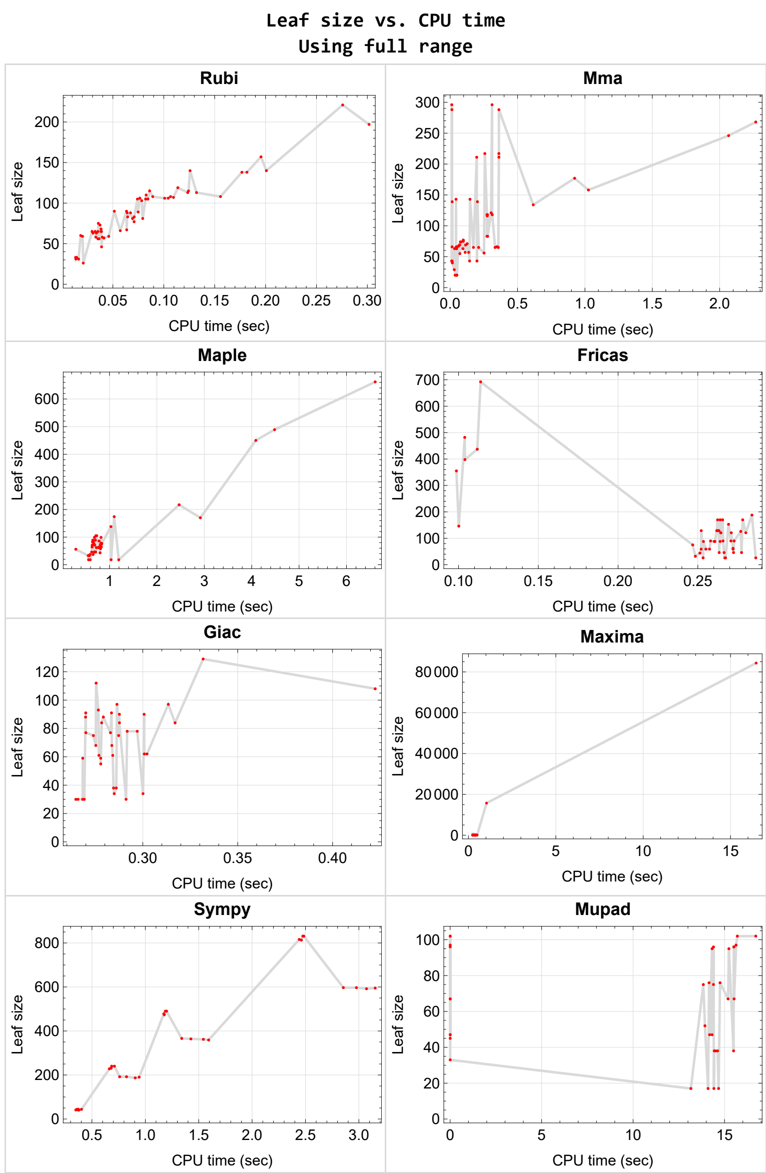


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {5}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

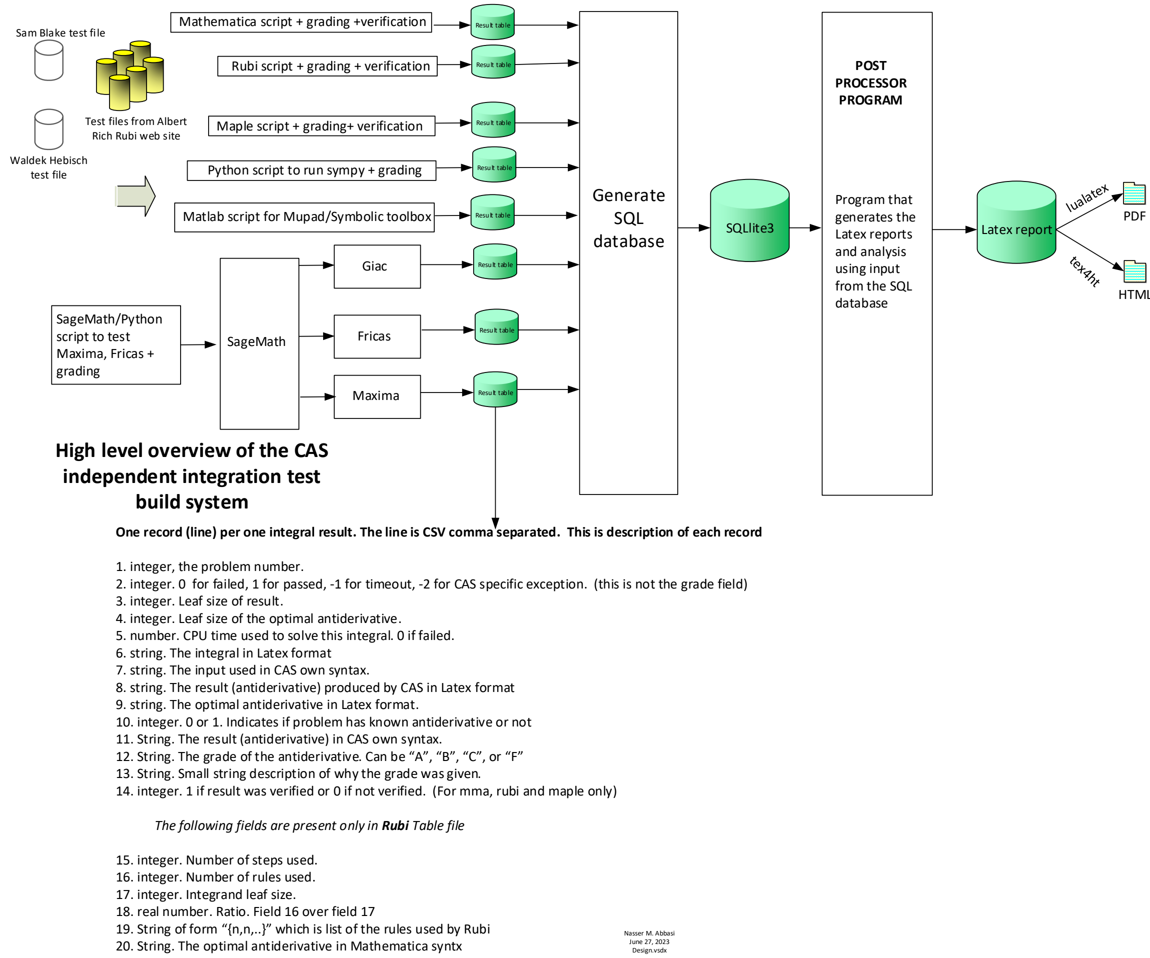
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 22
2.2 Detailed conclusion table per each integral for all CAS systems 25
2.3 Detailed conclusion table specific for Rubi results 38

2.1 List of integrals sorted by grade for each CAS

Rubi 22
Mma 22
Maple 23
Fricas 23
Maxima 23
Giac 24
Mupad 24
Sympy 24

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 42, 43, 44, 46, 47, 48, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 62 }

B grade { 37, 41, 45, 49, 56, 61 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 7, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 54, 55 }

B grade { 6, 50, 51, 52 }

C grade { 5, 53 }

F normal fail { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 56, 57, 58, 59, 60, 61, 62 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

B grade { 6, 7 }

C grade { 50, 51, 52, 53, 54, 55 }

F normal fail { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 56, 57, 58, 59, 60, 61, 62 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

B grade { 5, 6, 7 }

C grade { }

F normal fail { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 6, 7, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

B grade { 5 }

C grade { }

F normal fail { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 4, 5, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 53 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

F(-2) exception fail { }

Sympy

A grade { 18, 22, 26, 30, 34, 38, 42, 46 }

B grade { 35, 36, 37, 39, 40, 41, 43, 44, 45, 47, 48, 49 }

C grade { 19, 20, 21, 23, 24, 25, 27, 28, 29, 31, 32, 33 }

F normal fail { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

F(-1) timedout fail { 1 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	83	86	77	75	0	108	0
N.S.	1	1.00	0.70	0.72	0.65	0.63	0.00	0.91	0.00
time (sec)	N/A	0.114	0.273	0.779	0.465	0.247	0.000	0.422	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	71	73	60	62	0	84	0
N.S.	1	1.00	0.80	0.82	0.67	0.70	0.00	0.94	0.00
time (sec)	N/A	0.075	0.126	0.808	0.497	0.272	0.000	0.317	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	55	58	38	44	0	55	0
N.S.	1	1.00	0.93	0.98	0.64	0.75	0.00	0.93	0.00
time (sec)	N/A	0.046	0.072	0.810	0.469	0.251	0.000	0.278	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	29	43	20	32	0	30	33
N.S.	1	1.00	1.12	1.65	0.77	1.23	0.00	1.15	1.27
time (sec)	N/A	0.021	0.031	0.800	0.436	0.248	0.000	0.265	0.003

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	46	46	40	56	90	126	0	93	45
N.S.	1	1.00	0.87	1.22	1.96	2.74	0.00	2.02	0.98
time (sec)	N/A	0.039	0.016	0.289	0.425	0.277	0.000	0.277	0.002

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	63	138	15721	153	0	112	0
N.S.	1	1.00	0.82	1.79	204.17	1.99	0.00	1.45	0.00
time (sec)	N/A	0.071	0.095	1.030	1.034	0.269	0.000	0.275	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	65	174	84332	188	0	129	0
N.S.	1	1.00	0.61	1.63	788.15	1.76	0.00	1.21	0.00
time (sec)	N/A	0.110	0.211	1.099	16.466	0.284	0.000	0.332	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	69	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.115	0.000	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	69	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.070	0.000	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	69	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	0.070	0.000	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	67	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	0.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	67	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	69	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	0.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	74	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	0.076	0.000	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	77	0	0	0	0	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	0.097	0.000	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	74	0	0	0	0	0	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.081	0.000	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	18	24	26	41	30	38
N.S.	1	1.00	0.65	0.58	0.77	0.84	1.32	0.97	1.23
time (sec)	N/A	0.014	0.048	1.033	0.362	0.253	0.373	0.266	14.619

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	43	46	68	59	187	59	67
N.S.	1	1.00	0.77	0.82	1.21	1.05	3.34	1.05	1.20
time (sec)	N/A	0.036	0.144	0.679	0.331	0.257	0.905	0.278	15.523

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	56	62	111	90	359	75	96
N.S.	1	1.00	0.69	0.77	1.37	1.11	4.43	0.93	1.19
time (sec)	N/A	0.069	0.251	0.723	0.341	0.271	1.593	0.287	15.510

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	66	75	151	121	592	88	96
N.S.	1	1.00	0.62	0.71	1.42	1.14	5.58	0.83	0.91
time (sec)	N/A	0.101	0.345	0.834	0.332	0.280	3.071	0.279	15.507

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	20	18	24	26	41	30	38
N.S.	1	1.00	0.61	0.55	0.73	0.79	1.24	0.91	1.15
time (sec)	N/A	0.013	0.050	1.195	0.373	0.267	0.379	0.269	15.494

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	43	46	69	59	192	61	67
N.S.	1	1.00	0.74	0.79	1.19	1.02	3.31	1.05	1.16
time (sec)	N/A	0.040	0.199	0.687	0.411	0.272	0.824	0.277	15.194

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	65	64	112	90	364	77	95
N.S.	1	1.00	0.78	0.77	1.35	1.08	4.39	0.93	1.14
time (sec)	N/A	0.071	0.357	0.827	0.374	0.273	1.427	0.283	15.237

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	66	77	152	121	597	90	97
N.S.	1	1.00	0.61	0.71	1.41	1.12	5.53	0.83	0.90
time (sec)	N/A	0.107	0.351	0.838	0.347	0.271	2.854	0.288	15.630

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	20	18	24	26	42	30	38
N.S.	1	1.00	0.61	0.55	0.73	0.79	1.27	0.91	1.15
time (sec)	N/A	0.014	0.037	0.593	0.346	0.267	0.368	0.268	14.440

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	43	46	69	59	192	61	67
N.S.	1	1.00	0.74	0.79	1.19	1.02	3.31	1.05	1.16
time (sec)	N/A	0.033	0.011	0.623	0.323	0.255	0.759	0.284	0.002

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	65	64	112	90	366	77	96
N.S.	1	1.00	0.78	0.77	1.35	1.08	4.41	0.93	1.16
time (sec)	N/A	0.064	0.173	0.794	0.351	0.266	1.341	0.270	14.396

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	66	77	152	121	597	90	97
N.S.	1	1.00	0.61	0.71	1.41	1.12	5.53	0.83	0.90
time (sec)	N/A	0.156	0.047	0.669	0.401	0.264	2.977	0.301	0.002

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	18	24	26	44	30	38
N.S.	1	1.00	0.65	0.58	0.77	0.84	1.42	0.97	1.23
time (sec)	N/A	0.016	0.045	0.558	0.391	0.286	0.371	0.291	14.507

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	43	46	68	59	190	59	67
N.S.	1	1.00	0.77	0.82	1.21	1.05	3.39	1.05	1.20
time (sec)	N/A	0.035	0.015	0.709	0.379	0.252	0.942	0.268	0.002

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	65	62	111	90	362	75	95
N.S.	1	1.00	0.80	0.77	1.37	1.11	4.47	0.93	1.17
time (sec)	N/A	0.079	0.333	0.763	0.366	0.258	1.544	0.274	14.320

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	66	75	151	121	595	88	96
N.S.	1	1.00	0.62	0.71	1.42	1.14	5.61	0.83	0.91
time (sec)	N/A	0.104	0.014	0.821	0.331	0.264	3.153	0.270	0.003

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	33	48	46	41	34	17
N.S.	1	1.00	1.00	0.51	0.74	0.71	0.63	0.52	0.26
time (sec)	N/A	0.033	0.047	0.575	0.278	0.272	0.349	0.285	14.418

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	143	64	91	88	228	62	47
N.S.	1	1.00	1.59	0.71	1.01	0.98	2.53	0.69	0.52
time (sec)	N/A	0.063	0.147	0.632	0.272	0.254	0.663	0.301	14.183

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	217	83	135	129	474	78	76
N.S.	1	1.00	1.89	0.72	1.17	1.12	4.12	0.68	0.66
time (sec)	N/A	0.124	0.257	0.655	0.272	0.262	1.178	0.292	14.754

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	296	99	174	170	813	91	102
N.S.	1	1.00	2.11	0.71	1.24	1.21	5.81	0.65	0.73
time (sec)	N/A	0.201	0.311	0.821	0.245	0.262	2.462	0.284	16.708

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	37	50	46	44	38	17
N.S.	1	1.00	1.00	0.59	0.79	0.73	0.70	0.60	0.27
time (sec)	N/A	0.030	0.048	0.588	0.244	0.266	0.359	0.286	14.674

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	139	71	94	88	240	68	47
N.S.	1	1.00	1.58	0.81	1.07	1.00	2.73	0.77	0.53
time (sec)	N/A	0.064	0.203	0.635	0.275	0.260	0.711	0.284	14.320

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	211	89	137	129	490	84	76
N.S.	1	1.00	1.87	0.79	1.21	1.14	4.34	0.74	0.67
time (sec)	N/A	0.124	0.362	0.702	0.251	0.263	1.188	0.288	14.155

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	288	105	177	170	831	97	102
N.S.	1	1.00	2.09	0.76	1.28	1.23	6.02	0.70	0.74
time (sec)	N/A	0.182	0.362	0.730	0.237	0.278	2.475	0.286	15.699

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	37	50	46	44	38	17
N.S.	1	1.00	1.00	0.59	0.79	0.73	0.70	0.60	0.27
time (sec)	N/A	0.034	0.033	0.653	0.246	0.263	0.403	0.285	14.100

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	139	71	94	88	240	68	47
N.S.	1	1.00	1.58	0.81	1.07	1.00	2.73	0.77	0.53
time (sec)	N/A	0.067	0.015	0.687	0.273	0.264	0.684	0.275	0.002

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	211	89	137	129	490	84	75
N.S.	1	1.00	1.87	0.79	1.21	1.14	4.34	0.74	0.66
time (sec)	N/A	0.132	0.197	0.648	0.277	0.252	1.199	0.278	14.393

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	288	105	177	170	831	97	102
N.S.	1	1.00	2.09	0.76	1.28	1.23	6.02	0.70	0.74
time (sec)	N/A	0.177	0.013	0.712	0.251	0.266	2.484	0.313	0.002

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	33	48	46	42	34	17
N.S.	1	1.00	1.00	0.51	0.74	0.71	0.65	0.52	0.26
time (sec)	N/A	0.030	0.043	0.551	0.235	0.277	0.377	0.300	13.159

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	143	64	91	88	231	62	47
N.S.	1	1.00	1.59	0.71	1.01	0.98	2.57	0.69	0.52
time (sec)	N/A	0.051	0.044	0.779	0.261	0.261	0.680	0.302	0.002

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	217	83	135	129	478	78	75
N.S.	1	1.00	1.89	0.72	1.17	1.12	4.16	0.68	0.65
time (sec)	N/A	0.086	0.362	0.641	0.277	0.263	1.173	0.297	13.840

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	296	99	174	170	816	91	102
N.S.	1	1.00	2.11	0.71	1.24	1.21	5.83	0.65	0.73
time (sec)	N/A	0.126	0.012	0.682	0.247	0.264	2.442	0.270	0.002

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	177	662	0	437	0	0	0
N.S.	1	1.00	0.90	3.36	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.302	0.924	6.606	0.000	0.112	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	134	450	0	398	0	0	0
N.S.	1	1.00	0.85	2.87	0.00	2.54	0.00	0.00	0.00
time (sec)	N/A	0.196	0.617	4.087	0.000	0.104	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	170	0	355	0	0	0
N.S.	1	1.00	1.00	2.98	0.00	6.23	0.00	0.00	0.00
time (sec)	N/A	0.041	0.137	2.917	0.000	0.099	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	78	0	146	0	0	52
N.S.	1	1.00	1.00	1.37	0.00	2.56	0.00	0.00	0.91
time (sec)	N/A	0.041	0.111	0.821	0.000	0.100	0.000	0.000	13.929

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	83	217	0	482	0	0	0
N.S.	1	1.00	0.78	2.05	0.00	4.55	0.00	0.00	0.00
time (sec)	N/A	0.077	0.278	2.469	0.000	0.104	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	158	489	0	692	0	0	0
N.S.	1	1.00	0.71	2.21	0.00	3.13	0.00	0.00	0.00
time (sec)	N/A	0.276	1.026	4.483	0.000	0.114	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	246	0	0	0	0	0	0
N.S.	1	1.00	2.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.089	2.066	0.000	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	118	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	0.312	0.000	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	118	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	0.273	0.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	118	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	0.278	0.000	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	116	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	0.275	0.000	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	268	0	0	0	0	0	0
N.S.	1	1.00	2.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.083	2.268	0.000	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	121	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.078	0.304	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [50] had the largest ratio of [.5000000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	2	1.00	14	0.143
2	A	3	2	1.00	14	0.143
3	A	2	2	1.00	14	0.143
4	A	1	1	1.00	14	0.071
5	A	2	2	1.00	14	0.143
6	A	3	3	1.00	14	0.214
7	A	4	3	1.00	14	0.214
8	A	2	2	1.00	14	0.143
9	A	2	2	1.00	14	0.143
10	A	2	2	1.00	14	0.143
11	A	2	2	1.00	14	0.143
12	A	2	2	1.00	14	0.143
13	A	2	2	1.00	14	0.143
14	A	2	2	1.00	12	0.167
15	A	2	2	1.00	13	0.154
16	A	1	1	1.00	12	0.083
17	A	1	1	1.00	12	0.083
18	A	1	1	1.00	12	0.083
19	A	3	3	1.00	12	0.250
20	A	4	4	1.00	12	0.333
21	A	5	4	1.00	12	0.333
22	A	1	1	1.00	12	0.083
23	A	3	3	1.00	12	0.250
24	A	4	4	1.00	12	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	5	4	1.00	12	0.333
26	A	1	1	1.00	12	0.083
27	A	3	3	1.00	12	0.250
28	A	4	4	1.00	12	0.333
29	A	5	4	1.00	12	0.333
30	A	1	1	1.00	12	0.083
31	A	3	3	1.00	12	0.250
32	A	4	4	1.00	12	0.333
33	A	5	4	1.00	12	0.333
34	A	2	2	1.00	12	0.167
35	A	4	4	1.00	12	0.333
36	A	5	5	1.00	12	0.417
37	A	6	5	1.00	12	0.417
38	A	2	2	1.00	12	0.167
39	A	4	4	1.00	12	0.333
40	A	5	5	1.00	12	0.417
41	A	6	5	1.00	12	0.417
42	A	2	2	1.00	12	0.167
43	A	4	4	1.00	12	0.333
44	A	5	5	1.00	12	0.417
45	A	6	5	1.00	12	0.417
46	A	2	2	1.00	12	0.167
47	A	4	4	1.00	12	0.333
48	A	5	5	1.00	12	0.417
49	A	6	5	1.00	12	0.417
50	A	7	7	1.00	14	0.500
51	A	6	6	1.00	14	0.429
52	A	2	2	1.00	14	0.143
53	A	2	2	1.00	14	0.143
54	A	4	4	1.00	14	0.286
55	A	7	7	1.00	14	0.500
56	A	3	3	1.00	14	0.214
57	A	3	3	1.00	14	0.214
58	A	3	3	1.00	14	0.214
59	A	3	3	1.00	14	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	3	3	1.00	14	0.214
61	A	3	3	1.00	14	0.214
62	A	3	3	1.00	12	0.250

CHAPTER 3

LISTING OF INTEGRALS

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3.1 $\int (a+a\cos(c+dx))^{7/2}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 119

$$\int (a+a\cos(c+dx))^{7/2}\,dx = \frac{256a^4\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{64a^3\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{35d} + \frac{24a^2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2a(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d}$$

```
[Out] 24/35*a^2*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*a*(a+a*cos(d*x+c))^(5/2)*
sin(d*x+c)/d+256/35*a^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+64/35*a^3*sin(d
*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2726, 2725}

$$\int (a+a\cos(c+dx))^{7/2}\,dx = \frac{256a^4\sin(c+dx)}{35d\sqrt{a\cos(c+dx)+a}} + \frac{64a^3\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{35d} + \frac{24a^2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d} + \frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

```
[In] Int[(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (256*a^4*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (64*a^3*Sqrt[a + a
*Cos[c + d*x]]*Sin[c + d*x])/(35*d) + (24*a^2*(a + a*Cos[c + d*x])^(3/2)*Si
n[c + d*x])/(35*d) + (2*a*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d}+\frac{1}{7}(12a)\int(a+a\cos(c+dx))^{5/2}\,dx\\
&= \frac{24a^2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d}+\frac{2a(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d}\\
&\quad+\frac{1}{35}\left(96a^2\right)\int(a+a\cos(c+dx))^{3/2}\,dx\\
&= \frac{64a^3\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{35d}+\frac{24a^2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d}\\
&\quad+\frac{2a(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d}+\frac{1}{35}\left(128a^3\right)\int\sqrt{a+a\cos(c+dx)}\,dx\\
&= \frac{256a^4\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}}+\frac{64a^3\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{35d}\\
&\quad+\frac{24a^2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d}+\frac{2a(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$\int(a+a\cos(c+dx))^{7/2}\,dx=\frac{a^3\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(1225\sin\left(\frac{1}{2}(c+dx)\right)+245\sin\left(\frac{3}{2}(c+dx)\right)+49\sin\left(\frac{5}{2}\right.\right.}{140d}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(7/2),x]
```

[Out] (a^3*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(1225*Sin[(c + d*x)/2] + 245*Sin[(3*(c + d*x))/2] + 49*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*x))/2]))/(140*d)

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{16a^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(5\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+16\right)\sqrt{2}}{35\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, d}$	86

[In] int((a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)

[Out] 16/35*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(5*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d*x+1/2*c)^4+8*cos(1/2*d*x+1/2*c)^2+16)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int (a+a\cos(c+dx))^{7/2}\,dx=\frac{2\left(5\,a^{3}\cos\left(dx+c\right)^{3}+27\,a^{3}\cos\left(dx+c\right)^{2}+71\,a^{3}\cos\left(dx+c\right)+177\,a^{3}\right)\sqrt{a\cos\left(dx+c\right)+a}\,\mathrm{si}}{35\left(d\cos\left(dx+c\right)+d\right)}$$

[In] integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")

[Out] 2/35*(5*a^3*cos(d*x + c)^3 + 27*a^3*cos(d*x + c)^2 + 71*a^3*cos(d*x + c) + 177*a^3)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{7/2}\,dx=\text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(7/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.65

$$\int (a+a\cos(c+dx))^{7/2}\,dx = \frac{\left(5\sqrt{2}a^3\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+49\sqrt{2}a^3\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+245\sqrt{2}a^3\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+1225\sqrt{2}a^3\sin\right.}{140\,d}$$

[In] integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")

[Out] 1/140*(5*sqrt(2)*a^3*sin(7/2*d*x + 7/2*c) + 49*sqrt(2)*a^3*sin(5/2*d*x + 5/2*c) + 245*sqrt(2)*a^3*sin(3/2*d*x + 3/2*c) + 1225*sqrt(2)*a^3*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91

$$\int (a+a\cos(c+dx))^{7/2}\,dx = \frac{\sqrt{2}\left(5\,a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+49\,a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+2\right.}{140}$$

[In] integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="giac")

[Out] 1/140*sqrt(2)*(5*a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 49*a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 245*a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 1225*a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{7/2}\,dx = \int {\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}\,dx$$

[In] int((a + a*cos(c + d*x))^(7/2),x)

[Out] int((a + a*cos(c + d*x))^(7/2), x)

3.2 $\int (a+a\cos(c+dx))^{5/2}\,dx$

Optimal result 47
Rubi [A] (verified) 47
Mathematica [A] (verified) 48
Maple [A] (verified) 49
Fricas [A] (verification not implemented) 49
Sympy [F] 49
Maxima [A] (verification not implemented) 50
Giac [A] (verification not implemented) 50
Mupad [F(-1)] 50

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int (a+a\cos(c+dx))^{5/2}\,dx = \frac{64a^3\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{16a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

```
[Out] 2/5*a*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+64/15*a^3*sin(d*x+c)/d/(a+a*cos(d
*x+c))^(1/2)+16/15*a^2*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2726, 2725}

$$\int (a+a\cos(c+dx))^{5/2}\,dx = \frac{64a^3\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}} + \frac{16a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d} + \frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

```
[In] Int[(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a^2*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*(a + a*Cos[c + d*x])^(3/2)*Sin[c
+ d*x])/(5*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{1}{5}(8a)\int(a+a\cos(c+dx))^{3/2}\,dx\\
&= \frac{16a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}\\
&\quad+\frac{1}{15}(32a^2)\int\sqrt{a+a\cos(c+dx)}\,dx\\
&= \frac{64a^3\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{16a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d}\\
&\quad+\frac{2a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int(a+a\cos(c+dx))^{5/2}\,dx=\frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(150\sin\left(\frac{1}{2}(c+dx)\right)+25\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+\right.\right.}{30d}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2),x]
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(150*Sin[(c + d*x)/2] + 25
*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2]))/(30*d)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{8a^{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\right)\sqrt{2}}{15\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	73

```
[In] int((a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 8/15*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(3*cos(1/2*d*x+1/2*c)^4+4*co
s(1/2*d*x+1/2*c)^2+8)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

$$\int (a+a\cos(c+dx))^{5/2}\,dx=\frac{2\left(3\,a^{2}\cos\left(dx+c\right)^{2}+14\,a^{2}\cos\left(dx+c\right)+43\,a^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
[Out] 2/15*(3*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 43*a^2)*sqrt(a*cos(d*x +
 c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int (a+a\cos(c+dx))^{5/2}\,dx=\int\left(a\cos\left(c+dx\right)+a\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))**(5/2),x)
[Out] Integral((a*cos(c + d*x) + a)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int (a + a\cos(c + dx))^{5/2}\,dx = \frac{\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx + \frac{5}{2}\,c\right) + 25\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx + \frac{3}{2}\,c\right) + 150\sqrt{2}a^2\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int (a + a\cos(c + dx))^{5/2}\,dx = \frac{\sqrt{2}\left(3\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx + \frac{5}{2}\,c\right) + 25\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx + \frac{3}{2}\,c\right) + 1\right.}{30\,d}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] 1/30*sqrt(2)*(3*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 25*a^2
*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 150*a^2*sgn(cos(1/2*d*x +
 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}\,dx = \int (a + a\,\cos(c + d\,x))^{5/2}\,dx$$

```
[In] int((a + a*cos(c + d*x))^(5/2),x)
[Out] int((a + a*cos(c + d*x))^(5/2), x)
```

3.3 $\int (a+a\cos(c+dx))^{3/2}\,dx$

Optimal result . . . 51
Rubi [A] (verified) . . . 51
Mathematica [A] (verified) . . . 52
Maple [A] (verified) . . . 52
Fricas [A] (verification not implemented) . . . 53
Sympy [F] . . . 53
Maxima [A] (verification not implemented) . . . 53
Giac [A] (verification not implemented) . . . 54
Mupad [F(-1)] . . . 54

Optimal result

Integrand size = 14, antiderivative size = 59

$$\int (a+a\cos(c+dx))^{3/2}\,dx=\frac{8a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2a\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}$$

```
[Out] 8/3*a^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*a*sin(d*x+c)*(a+a*cos(d*x+c
))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2726, 2725}

$$\int (a+a\cos(c+dx))^{3/2}\,dx=\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

```
[In] Int[(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c
 + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}(4a)\int\sqrt{a+a\cos(c+dx)}\,dx \\
&= \frac{8a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int(a+a\cos(c+dx))^{3/2}\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(9\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)}{3d}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2),x]
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(9*Sin[(c + d*x)/2] + Sin[(3
*(c + d*x))/2]))/(3*d)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{4a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)\sqrt{2}}{3\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	58

```
[In] int((a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 4/3*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(cos(1/2*d*x+1/2*c)^2+2)*2^(1
/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int (a + a\cos(c + dx))^{3/2}\, dx = \frac{2\left(a\cos\left(dx + c\right) + 5\,a\right)\sqrt{a\cos\left(dx + c\right) + a}\sin\left(dx + c\right)}{3\left(d\cos\left(dx + c\right) + d\right)}$$

```
[In] integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a*cos(d*x + c) + 5*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x
 + c) + d)
```

Sympy [F]

$$\int (a + a\cos(c + dx))^{3/2}\, dx = \int \left(a\cos\left(c + dx\right) + a\right)^{\frac{3}{2}}\, dx$$

```
[In] integrate((a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*cos(c + d*x) + a)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int (a + a\cos(c + dx))^{3/2}\, dx = \frac{\left(\sqrt{2}a\sin\left(\frac{3}{2}\,dx + \frac{3}{2}\,c\right) + 9\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

```
[In] integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqr
t(a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int (a+a\cos(c+dx))^{3/2}\,dx = \frac{\sqrt{2}\left(a\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,a\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

```
[In] integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] 1/3*sqrt(2)*(a*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 9*a*sgn(cos
(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\,dx = \int \left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

```
[In] int((a + a*cos(c + d*x))^(3/2),x)
[Out] int((a + a*cos(c + d*x))^(3/2), x)
```

3.4 $\int \sqrt{a+a\cos(c+dx)}\,dx$

Optimal result 55
Rubi [A] (verified) 55
Mathematica [A] (verified) 56
Maple [A] (verified) 56
Fricas [A] (verification not implemented) 56
Sympy [F] 57
Maxima [A] (verification not implemented) 57
Giac [A] (verification not implemented) 57
Mupad [B] (verification not implemented) 57

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \sqrt{a+a\cos(c+dx)}\,dx = \frac{2a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

```
[Out] 2*a*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2725}

$$\int \sqrt{a+a\cos(c+dx)}\,dx = \frac{2a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[In] Int[Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \sqrt{a+a\cos(c+dx)}\,dx=\frac{2\sqrt{a(1+\cos(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]],x]
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/d
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

method	result	size
default	$\frac{2a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}}{\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	43
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(e^{i(dx+c)}+1\right)^2 e^{-i(dx+c)}}\,\left(e^{i(dx+c)}-1\right)}{\left(e^{i(dx+c)}+1\right)d}$	60

```
[In] int((a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^
(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \sqrt{a+a\cos(c+dx)}\,dx=\frac{2\,\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)+d}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \sqrt{a+a\cos(c+dx)}\,dx = \int \sqrt{a\cos\left(c+dx\right)+a}\,dx$$

[In] integrate((a+a*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(a*cos(c + d*x) + a), x)

Maxima [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \sqrt{a+a\cos(c+dx)}\,dx = \frac{2\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \sqrt{a+a\cos(c+dx)}\,dx = \frac{2\sqrt{2}\sqrt{a}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] 2*sqrt(2)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \sqrt{a+a\cos(c+dx)}\,dx = \frac{2\,\sin\left(c+d\,x\right)\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}}{d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

[In] int((a + a*cos(c + d*x))^(1/2),x)

[Out] (2*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(d*(cos(c + d*x) + 1))

3.5 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result 58
Rubi [A] (verified) 58
Mathematica [A] (verified) 59
Maple [C] (warning: unable to verify) 59
Fricas [A] (verification not implemented) 60
Sympy [F] 60
Maxima [B] (verification not implemented) 60
Giac [B] (verification not implemented) 61
Mupad [B] (verification not implemented) 61

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^
(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2728, 212}

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[In] Int[1/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\\ &= \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

```
[In] Integrate[1/Sqrt[a + a*Cos[c + d*x]],x]
[Out] (2*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])
])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\sqrt{2}\,\text{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle|1\right)}{d\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\text{csgn}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	56

```
[In] int(1/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/d*2^(1/2)/sec(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/csgn(cos(1/2*
d*x+1/2*c))*InverseJacobiAM(1/2*d*x+1/2*c,1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.74

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2\sqrt{a}d}, -\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}}{\sin(dx+c)}\right)}{d}\right]$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)
*sqrt(-1/a)/sin(d*x + c))/d]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cos(c+dx)+a}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(a*cos(c + d*x) + a), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.

Time = 0.43 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}\right.\right.}{2\sqrt{a}d}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.02

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)}{\sqrt{a}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{\sqrt{a}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{4\,d}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c) + 2))/(s
qrt(a)*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c)
+ sin(1/2*d*x + 1/2*c) - 2))/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,(a+a\,\cos(c+d\,x))}{a}}}{d\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}$$

```
[In] int(1/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (ellipticF(c/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2))/(d*(a + a*
cos(c + d*x))^(1/2))
```

3.6 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result 62
Rubi [A] (verified) 62
Mathematica [A] (verified) 63
Maple [B] (verified) 63
Fricas [B] (verification not implemented) 64
Sympy [F] 64
Maxima [B] (verification not implemented) 65
Giac [A] (verification not implemented) 75
Mupad [F(-1)] 75

Optimal result

Integrand size = 14, antiderivative size = 77

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

```
[Out] 1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*
2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2729, 2728, 212}

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[In] Int[(a + a*Cos[c + d*x])^(-3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]/(2*Sqrt[
2]*a^(3/2)*d) + Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a}\\
&= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}-\frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{2ad}\\
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\operatorname{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)+\tan\left(\frac{1}{2}(c+dx\right.\right.}{d(a(1+\cos(c+dx)))^{3/2}}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(-3/2),x]
[Out] (Cos[(c + d*x)/2]^2*(ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + Tan[(c +
d*x)/2]))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(62) = 124.

Time = 1.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.79

method	result	size
default	$\frac{\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\right)}{4a^{\frac{5}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	138

```
[In] int(1/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^(5/2)/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2^(1/2)*ln(2
*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1
/2*d*x+1/2*c)^2+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x
+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(62) = 124$.

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.99

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{}}{\cos(dx+c)^{2}+2\,\cos}\right.}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(\right.\right.}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x
+ c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(
d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt(a*cos(d*x +
 c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d
)
```

Sympy [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a\cos\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*cos(c + d*x) + a)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15721 vs. $2(62) = 124$.

Time = 1.03 (sec) , antiderivative size = 15721, normalized size of antiderivative = 204.17

$$\int \frac{1}{(a + a\cos(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(32*(cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d
*x + 3/2*c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(
d*x + c))*cos(3*d*x + 3*c)^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) +
3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x
+ 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3
/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3
*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1
)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*
x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(2/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 32*(cos(3/2*d*x
+ 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + cos(d*x
 + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(3*d*x +
 3*c)^2 + 32*(6*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 9
6*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 96*sin(2*d*x + 2*c)^2*sin(3/2*d
*x + 3/2*c) + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3
/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*
d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) +
 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*c
os(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3
/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*
d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(2/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*(2*(3*cos(d*x + c) + 1)*cos(2*
d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c)
 + 3*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 2*(3*sin(3/2*d*x + 3/2*c)*si
n(d*x + c) + cos(3/2*d*x + 3/2*c))*sin(2*d*x + 2*c) + (3*cos(d*x + c)^2 + 3
*sin(d*x + c)^2 + 2*cos(d*x + c))*sin(3/2*d*x + 3/2*c) + 2*cos(3/2*d*x + 3/
2*c)*sin(d*x + c))*cos(3*d*x + 3*c) - 4*(6*(sin(2*d*x + 2*c) + sin(d*x + c)
)*sin(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^3 + (2*(3*cos(2*d*x + 2*c) + 3*cos(
d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1
)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 9*sin(2*d*x
+ 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x
+ c) + 1)*sin(3*d*x + 3*c) + 3*(2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)
*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x +
```

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 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin(2*d*x + 2*c) + sin
(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*sin(2*d*x + 2*c)^2 + 1
8*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*si
n(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*(2*(3*cos(2*
d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*
(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x +
c)^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3
*c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x
 + c)^2 + 6*cos(d*x + c) + 1)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 4*(8*cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*cos(2*d*x + 2*c)^2*s
in(3/2*d*x + 3/2*c) - 144*cos(2*d*x + 2*c)*cos(d*x + c)*sin(3/2*d*x + 3/2*c
) - 8*sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*sin(2*d*x + 2*c)^2*sin(3
/2*d*x + 3/2*c) - 16*(3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 3*cos(3/2*d
*x + 3/2*c)*sin(d*x + c) - sin(3/2*d*x + 3/2*c))*cos(3*d*x + 3*c) - 48*(cos
(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c
) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*
x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*
c)*sin(d*x + c))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 16*(cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c) + 3*sin(2*d*x + 2*c)*sin(3/2
*d*x + 3/2*c) + 3*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x + 3/2*c))
*sin(3*d*x + 3*c) - 48*(3*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x +
 3/2*c))*sin(2*d*x + 2*c) - 8*(9*cos(d*x + c)^2 + 9*sin(d*x + c)^2 - 1)*sin
(3/2*d*x + 3/2*c) - 48*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 3*(2*(3*cos(2*d*
x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3
*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)
^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c
)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x +
 c)^2 + 6*cos(d*x + c) + 1)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 - 4*(8*cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*cos(2*d*x + 2*c)^2*sin
(3/2*d*x + 3/2*c) - 144*cos(2*d*x + 2*c)*cos(d*x + c)*sin(3/2*d*x + 3/2*c)
- 8*sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*sin(2*d*x + 2*c)^2*sin(3/2
*d*x + 3/2*c) - 16*(3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 3*cos(3/2*d*x
 + 3/2*c)*sin(d*x + c) - sin(3/2*d*x + 3/2*c))*cos(3*d*x + 3*c) - 16*(cos(3
*d*x + 3*c)*cos(3/2*d*x + 3/2*c) + 3*sin(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c)
+ 3*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x + 3/2*c))*sin(3*d*x + 3
*c) - 48*(3*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x + 3/2*c))*sin(2
*d*x + 2*c) - 8*(9*cos(d*x + c)^2 + 9*sin(d*x + c)^2 - 1)*sin(3/2*d*x + 3/2
*c) - 48*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 3*(2*(3*cos(2*d*x + 2*c) + 3*c
os(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c)
+ 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin(2*
d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*sin(2*
d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(
d*x + c) + 1)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))
```

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*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*(6*(sin(2
*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^3 + (2*(3
*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)
^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos
(d*x + c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*s
in(d*x + c)^2 + 6*cos(d*x + c) + 1)*sin(3*d*x + 3*c))*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + (2*(3*cos(2*d*x + 2*c) + 3*cos(d*
x + c) + 2)*cos(3*d*x + 3*c)^3 + cos(3*d*x + 3*c)^4 + 6*(sin(2*d*x + 2*c) +
 sin(d*x + c))*sin(3*d*x + 3*c)^3 + sin(3*d*x + 3*c)^4 + 3*(6*(cos(d*x + c)
 + 1)*cos(2*d*x + 2*c) + 3*cos(2*d*x + 2*c)^2 + 3*cos(d*x + c)^2 + 3*sin(2*
d*x + 2*c)^2 + 6*sin(2*d*x + 2*c)*sin(d*x + c) + 3*sin(d*x + c)^2 + 6*cos(d
*x + c) + 2)*cos(3*d*x + 3*c)^2 + 9*(2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c)
 + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*
d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin(2*d*x + 2*c)
+ sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*sin(2*d*x + 2*c)^
2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) +
1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*(2*(3
*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)
^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos
(d*x + c)^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*
d*x + 3*c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*
sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 + (2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 2)*cos
(3*d*x + 3*c) + 2*cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2
*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*s
in(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 2)*sin(3
*d*x + 3*c)^2 + 9*(2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x +
3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos
(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*si
n(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x +
 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*sin(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*(2*(3*cos(2*d*x + 2*c)
+ 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x
+ c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(s
in(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*s
in(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6
*cos(d*x + c) + 1)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*(9*(2*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2
+ 9*cos(d*x + c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c
) + 9*sin(d*x + c)^2 + 9*cos(d*x + c) + 2)*cos(3*d*x + 3*c) + 6*(3*cos(d*x
+ c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(3
*(2*cos(2*d*x + 2*c) + 2*cos(d*x + c) + 1)*cos(3*d*x + 3*c)^2 + cos(3*d*x +
 3*c)^3 + (cos(3*d*x + 3*c) + 1)*sin(3*d*x + 3*c)^2 + 3*(2*(3*cos(d*x + c)
+ 2)*cos(2*d*x + 2*c) + 3*cos(2*d*x + 2*c)^2 + 3*cos(d*x + c)^2 + 3*sin(2*d
*x + 2*c)^2 + 6*sin(2*d*x + 2*c)*sin(d*x + c) + 3*sin(d*x + c)^2 + 4*cos(d*
```

```
x + c) + 1)*cos(3*d*x + 3*c) + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*
cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 3*(2*(3*cos(2*d*x + 2*c) + 3*cos(d*
x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*
cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin(2*d*x +
 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*sin(2*d*x +
 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x +
 c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(
(sin(2*d*x + 2*c) + sin(d*x + c))*cos(3*d*x + 3*c) + sin(2*d*x + 2*c) + sin
(d*x + c))*sin(3*d*x + 3*c) + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*si
n(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*cos(4/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(3*(2*cos(2*d*x + 2*c) + 2*cos(d*
x + c) + 1)*cos(3*d*x + 3*c)^2 + cos(3*d*x + 3*c)^3 + (cos(3*d*x + 3*c) + 1
)*sin(3*d*x + 3*c)^2 + 3*(2*(3*cos(d*x + c) + 2)*cos(2*d*x + 2*c) + 3*cos(2
*d*x + 2*c)^2 + 3*cos(d*x + c)^2 + 3*sin(2*d*x + 2*c)^2 + 6*sin(2*d*x + 2*c
)*sin(d*x + c) + 3*sin(d*x + c)^2 + 4*cos(d*x + c) + 1)*cos(3*d*x + 3*c) +
6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x
+ c)^2 + 6*((sin(2*d*x + 2*c) + sin(d*x + c))*cos(3*d*x + 3*c) + sin(2*d*x
+ 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d
*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*cos(2/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*((sin(2*d*x + 2*c) +
sin(d*x + c))*cos(3*d*x + 3*c)^2 + 2*(sin(2*d*x + 2*c) + sin(d*x + c))*cos(
3*d*x + 3*c) + sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 9*sin(2*
d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*(6*(
sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^3 +
(2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x +
 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 +
9*cos(d*x + c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c)
+ 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*sin(3*d*x + 3*c) + 3*(2*(3*cos(2*d
*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(
3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c
)^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*
c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x
+ c)^2 + 6*cos(d*x + c) + 1)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 6*(6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c)^2 + sin(3*d*x +
 3*c)^3 + (2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + c
os(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x +
 2*c)^2 + 9*cos(d*x + c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin
(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*sin(3*d*x + 3*c))*sin(2/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*cos(d*x + c) + 1
)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - (2*(3*cos(2*d*x + 2
*c) + 3*cos(d*x + c) + 2)*cos(3*d*x + 3*c)^3 + cos(3*d*x + 3*c)^4 + 6*(sin(
2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c)^3 + sin(3*d*x + 3*c)^4 + 3*(6
```

```
*(cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 3*cos(2*d*x + 2*c)^2 + 3*cos(d*x + c
)^2 + 3*sin(2*d*x + 2*c)^2 + 6*sin(2*d*x + 2*c)*sin(d*x + c) + 3*sin(d*x +
c)^2 + 6*cos(d*x + c) + 2)*cos(3*d*x + 3*c)^2 + 9*(2*(3*cos(2*d*x + 2*c) +
3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x +
c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin
(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*sin
(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*c
os(d*x + c) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 9*(2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + co
s(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x +
2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x +
 3*c) + sin(3*d*x + 3*c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin
(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*cos(2/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + (2*(3*cos(2*d*x + 2*c) + 3*cos(d*x
 + c) + 2)*cos(3*d*x + 3*c) + 2*cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)
*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 9*sin(2*d*x +
 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x +
 c) + 2)*sin(3*d*x + 3*c)^2 + 9*(2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1
)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x
+ 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin(2*d*x + 2*c) + si
n(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*sin(2*d*x + 2*c)^2 +
18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*s
in(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*(2*(3*cos
(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 +
 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x
 + c)^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x
+ 3*c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(
d*x + c)^2 + 6*cos(d*x + c) + 1)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c)))^2 + 2*(9*(2*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2
*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*
c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 9*cos(d*x + c) + 2)*cos(3*d*x + 3*c) +
 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x
 + c)^2 + 6*(3*(2*cos(2*d*x + 2*c) + 2*cos(d*x + c) + 1)*cos(3*d*x + 3*c)^2
 + cos(3*d*x + 3*c)^3 + (cos(3*d*x + 3*c) + 1)*sin(3*d*x + 3*c)^2 + 3*(2*(3
*cos(d*x + c) + 2)*cos(2*d*x + 2*c) + 3*cos(2*d*x + 2*c)^2 + 3*cos(d*x + c)
^2 + 3*sin(2*d*x + 2*c)^2 + 6*sin(2*d*x + 2*c)*sin(d*x + c) + 3*sin(d*x + c
)^2 + 4*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 6*(3*cos(d*x + c) + 1)*cos(2*d
*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 3*(2*(3*cos(2*d*x + 2
*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(
d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 +
6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 +
 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2
 + 6*cos(d*x + c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 6*((sin(2*d*x + 2*c) + sin(d*x + c))*cos(3*d*x + 3*c) + sin(2*d*
x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 9*sin(2*d*x + 2*c)^2 + 18*sin(2
```

```
*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*cos(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(3*(2*cos(2*d*x + 2
*c) + 2*cos(d*x + c) + 1)*cos(3*d*x + 3*c)^2 + cos(3*d*x + 3*c)^3 + (cos(3*
d*x + 3*c) + 1)*sin(3*d*x + 3*c)^2 + 3*(2*(3*cos(d*x + c) + 2)*cos(2*d*x +
2*c) + 3*cos(2*d*x + 2*c)^2 + 3*cos(d*x + c)^2 + 3*sin(2*d*x + 2*c)^2 + 6*s
in(2*d*x + 2*c)*sin(d*x + c) + 3*sin(d*x + c)^2 + 4*cos(d*x + c) + 1)*cos(3
*d*x + 3*c) + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^
2 + 9*cos(d*x + c)^2 + 6*((sin(2*d*x + 2*c) + sin(d*x + c))*cos(3*d*x + 3*c
) + sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 9*sin(2*d*x + 2*c)^
2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) +
1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*((sin(2
*d*x + 2*c) + sin(d*x + c))*cos(3*d*x + 3*c)^2 + 2*(sin(2*d*x + 2*c) + sin(
d*x + c))*cos(3*d*x + 3*c) + sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3
*c) + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x +
 c)^2 + 6*(6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c)^2 + sin(3*d
*x + 3*c)^3 + (2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c)
 + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d
*x + 2*c)^2 + 9*cos(d*x + c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)
*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*sin(3*d*x + 3*c) + 3
*(2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x
+ 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 +
 9*cos(d*x + c)^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) +
sin(3*d*x + 3*c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c
) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*sin(2/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 6*(6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c)^2
 + sin(3*d*x + 3*c)^3 + (2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*
d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) +
 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*
d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*sin(3*d*x
+ 3*c))*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*co
s(d*x + c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 -
2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 32*(2
*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*cos(3/2*d*x + 3/2*c) + 3*cos(2*d*x +
 2*c)^2cos(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c)^2 +
2*(cos(2*d*x + 2*c)*cos(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*cos(d*x + c
) - sin(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - sin(3/2*d*x + 3/2*c)*sin(d*x +
c))*cos(3*d*x + 3*c) + (3*cos(d*x + c)^2 + 3*sin(d*x + c)^2 + 2*cos(d*x + c
))*cos(3/2*d*x + 3/2*c) + 2*(3*cos(3/2*d*x + 3/2*c)*sin(d*x + c) - sin(3/2*
d*x + 3/2*c))*sin(2*d*x + 2*c) - 2*sin(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(3
*d*x + 3*c) + 32*(6*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x + 3/2*c
))*sin(2*d*x + 2*c) + 32*(3*cos(d*x + c)^2 + 3*sin(d*x + c)^2 + cos(d*x + c
))*sin(3/2*d*x + 3/2*c) + 32*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 4*(3*(2*co
s(2*d*x + 2*c) + 2*cos(d*x + c) + 1)*cos(3*d*x + 3*c)^2 + cos(3*d*x + 3*c)^
```

```
3 + (cos(3*d*x + 3*c) + 1)*sin(3*d*x + 3*c)^2 + 3*(2*(3*cos(d*x + c) + 2)*c
os(2*d*x + 2*c) + 3*cos(2*d*x + 2*c)^2 + 3*cos(d*x + c)^2 + 3*sin(2*d*x + 2
*c)^2 + 6*sin(2*d*x + 2*c)*sin(d*x + c) + 3*sin(d*x + c)^2 + 4*cos(d*x + c)
 + 1)*cos(3*d*x + 3*c) + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*
d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 3*(2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c)
 + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*
d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin(2*d*x + 2*c)
+ sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*sin(2*d*x + 2*c)^
2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) +
1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*(2*(3*c
os(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2
 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d
*x + c)^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*
x + 3*c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*si
n(d*x + c)^2 + 6*cos(d*x + c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 6*((sin(2*d*x + 2*c) + sin(d*x + c))*cos(3*d*x + 3*c
) + sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 9*sin(2*d*x + 2*c)^
2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) +
1)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 4*(8*cos(
3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c) + 48*(3*cos(d*x + c) + 1)*cos(2*d*x + 2
*c)*cos(3/2*d*x + 3/2*c) + 72*cos(2*d*x + 2*c)^2*cos(3/2*d*x + 3/2*c) - 8*c
os(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2 + 72*cos(3/2*d*x + 3/2*c)*sin(2*d*x
+ 2*c)^2 + 144*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c)*sin(d*x + c) + 16*((3*
cos(d*x + c) + 1)*cos(3/2*d*x + 3/2*c) + 3*cos(2*d*x + 2*c)*cos(3/2*d*x + 3
/2*c))*cos(3*d*x + 3*c) + 8*(9*cos(d*x + c)^2 + 9*sin(d*x + c)^2 + 6*cos(d*
x + c) + 1)*cos(3/2*d*x + 3/2*c) - 3*(2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c
) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2
*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin(2*d*x + 2*c)
 + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*sin(2*d*x + 2*c)
^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) +
 1)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*((3*c
os(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*
c) + 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c))*sin(3*d*x + 3*c) - 48*(cos(3/
2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) -
 (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x +
 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*
sin(d*x + c))*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))
*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 4*(8*cos(3*
d*x + 3*c)^2*cos(3/2*d*x + 3/2*c) + 48*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c
)*cos(3/2*d*x + 3/2*c) + 72*cos(2*d*x + 2*c)^2*cos(3/2*d*x + 3/2*c) - 8*cos
(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2 + 72*cos(3/2*d*x + 3/2*c)*sin(2*d*x +
2*c)^2 + 144*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c)*sin(d*x + c) + 16*((3*co
s(d*x + c) + 1)*cos(3/2*d*x + 3/2*c) + 3*cos(2*d*x + 2*c)*cos(3/2*d*x + 3/2
*c))*cos(3*d*x + 3*c) + 8*(9*cos(d*x + c)^2 + 9*sin(d*x + c)^2 + 6*cos(d*x
+ c) + 1)*cos(3/2*d*x + 3/2*c) - 3*(2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c)
```

```
+ 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d
*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin(2*d*x + 2*c) +
 sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*sin(2*d*x + 2*c)^2
 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1
)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*((3*cos
(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c)
 + 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c))*sin(3*d*x + 3*c))*sin(2/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 4*(3*(2*cos(2*d*x + 2*c)
+ 2*cos(d*x + c) + 1)*cos(3*d*x + 3*c)^2 + cos(3*d*x + 3*c)^3 + (cos(3*d*x
+ 3*c) + 1)*sin(3*d*x + 3*c)^2 + 3*(2*(3*cos(d*x + c) + 2)*cos(2*d*x + 2*c)
 + 3*cos(2*d*x + 2*c)^2 + 3*cos(d*x + c)^2 + 3*sin(2*d*x + 2*c)^2 + 6*sin(2
*d*x + 2*c)*sin(d*x + c) + 3*sin(d*x + c)^2 + 4*cos(d*x + c) + 1)*cos(3*d*x
 + 3*c) + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 +
9*cos(d*x + c)^2 + 6*((sin(2*d*x + 2*c) + sin(d*x + c))*cos(3*d*x + 3*c) +
sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 9*sin(2*d*x + 2*c)^2 +
18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/((sqrt(2)*a*co
s(3*d*x + 3*c)^4 + sqrt(2)*a*sin(3*d*x + 3*c)^4 + 2*(3*sqrt(2)*a*cos(2*d*x
+ 2*c) + 3*sqrt(2)*a*cos(d*x + c) + 2*sqrt(2)*a)*cos(3*d*x + 3*c)^3 + 6*(sq
rt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a*sin(d*x + c))*sin(3*d*x + 3*c)^3 + 9*s
qrt(2)*a*cos(2*d*x + 2*c)^2 + 9*sqrt(2)*a*cos(d*x + c)^2 + 9*sqrt(2)*a*sin(
2*d*x + 2*c)^2 + 18*sqrt(2)*a*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sqrt(2)*a*s
in(d*x + c)^2 + 3*(3*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 3*sqrt(2)*a*cos(d*x + c
)^2 + 3*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 6*sqrt(2)*a*sin(2*d*x + 2*c)*sin(d*x
 + c) + 3*sqrt(2)*a*sin(d*x + c)^2 + 6*sqrt(2)*a*cos(d*x + c) + 6*(sqrt(2)*
a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + 2*sqrt(2)*a)*cos(3*d*x + 3*c
)^2 + 9*(sqrt(2)*a*cos(3*d*x + 3*c)^2 + 9*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 9*
sqrt(2)*a*cos(d*x + c)^2 + sqrt(2)*a*sin(3*d*x + 3*c)^2 + 9*sqrt(2)*a*sin(2
*d*x + 2*c)^2 + 18*sqrt(2)*a*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sqrt(2)*a*si
n(d*x + c)^2 + 6*sqrt(2)*a*cos(d*x + c) + 2*(3*sqrt(2)*a*cos(2*d*x + 2*c) +
 3*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(3*d*x + 3*c) + 6*(3*sqrt(2)*a*co
s(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + 6*(sqrt(2)*a*sin(2*d*x + 2*c) +
sqrt(2)*a*sin(d*x + c))*sin(3*d*x + 3*c) + sqrt(2)*a)*cos(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*(sqrt(2)*a*cos(3*d*x + 3*c)^2
 + 9*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 9*sqrt(2)*a*cos(d*x + c)^2 + sqrt(2)*a*
sin(3*d*x + 3*c)^2 + 9*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 18*sqrt(2)*a*sin(2*d*
x + 2*c)*sin(d*x + c) + 9*sqrt(2)*a*sin(d*x + c)^2 + 6*sqrt(2)*a*cos(d*x +
c) + 2*(3*sqrt(2)*a*cos(2*d*x + 2*c) + 3*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a
)*cos(3*d*x + 3*c) + 6*(3*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2
*c) + 6*(sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a*sin(d*x + c))*sin(3*d*x + 3
*c) + sqrt(2)*a)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + (2*sqrt(2)*a*cos(3*d*x + 3*c)^2 + 9*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 9
*sqrt(2)*a*cos(d*x + c)^2 + 9*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 18*sqrt(2)*a*s
in(2*d*x + 2*c)*sin(d*x + c) + 9*sqrt(2)*a*sin(d*x + c)^2 + 6*sqrt(2)*a*cos
(d*x + c) + 2*(3*sqrt(2)*a*cos(2*d*x + 2*c) + 3*sqrt(2)*a*cos(d*x + c) + 2*
```

```
sqrt(2)*a)*cos(3*d*x + 3*c) + 6*(3*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(
2*d*x + 2*c) + 2*sqrt(2)*a)*sin(3*d*x + 3*c)^2 + 9*(sqrt(2)*a*cos(3*d*x + 3
*c)^2 + 9*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 9*sqrt(2)*a*cos(d*x + c)^2 + sqrt(
2)*a*sin(3*d*x + 3*c)^2 + 9*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 18*sqrt(2)*a*sin
(2*d*x + 2*c)*sin(d*x + c) + 9*sqrt(2)*a*sin(d*x + c)^2 + 6*sqrt(2)*a*cos(d
*x + c) + 2*(3*sqrt(2)*a*cos(2*d*x + 2*c) + 3*sqrt(2)*a*cos(d*x + c) + sqrt
(2)*a)*cos(3*d*x + 3*c) + 6*(3*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*
x + 2*c) + 6*(sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a*sin(d*x + c))*sin(3*d*
x + 3*c) + sqrt(2)*a)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 9*(sqrt(2)*a*cos(3*d*x + 3*c)^2 + 9*sqrt(2)*a*cos(2*d*x + 2*c)^
2 + 9*sqrt(2)*a*cos(d*x + c)^2 + sqrt(2)*a*sin(3*d*x + 3*c)^2 + 9*sqrt(2)*a
*sin(2*d*x + 2*c)^2 + 18*sqrt(2)*a*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sqrt(2
)*a*sin(d*x + c)^2 + 6*sqrt(2)*a*cos(d*x + c) + 2*(3*sqrt(2)*a*cos(2*d*x +
2*c) + 3*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(3*d*x + 3*c) + 6*(3*sqrt(2
)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + 6*(sqrt(2)*a*sin(2*d*x + 2
*c) + sqrt(2)*a*sin(d*x + c))*sin(3*d*x + 3*c) + sqrt(2)*a)*sin(2/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*sqrt(2)*a*cos(d*x + c)
+ 2*(9*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 9*sqrt(2)*a*cos(d*x + c)^2 + 9*sqrt(2
)*a*sin(2*d*x + 2*c)^2 + 18*sqrt(2)*a*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sqr
t(2)*a*sin(d*x + c)^2 + 9*sqrt(2)*a*cos(d*x + c) + 9*(2*sqrt(2)*a*cos(d*x +
 c) + sqrt(2)*a)*cos(2*d*x + 2*c) + 2*sqrt(2)*a)*cos(3*d*x + 3*c) + 6*(3*sq
rt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + 6*(sqrt(2)*a*cos(3*d*x
 + 3*c)^3 + 9*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 9*sqrt(2)*a*cos(d*x + c)^2 + 9
*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 18*sqrt(2)*a*sin(2*d*x + 2*c)*sin(d*x + c)
+ 9*sqrt(2)*a*sin(d*x + c)^2 + 3*(2*sqrt(2)*a*cos(2*d*x + 2*c) + 2*sqrt(2)*
a*cos(d*x + c) + sqrt(2)*a)*cos(3*d*x + 3*c)^2 + (sqrt(2)*a*cos(3*d*x + 3*c
) + sqrt(2)*a)*sin(3*d*x + 3*c)^2 + 6*sqrt(2)*a*cos(d*x + c) + 3*(3*sqrt(2)
*a*cos(2*d*x + 2*c)^2 + 3*sqrt(2)*a*cos(d*x + c)^2 + 3*sqrt(2)*a*sin(2*d*x
+ 2*c)^2 + 6*sqrt(2)*a*sin(2*d*x + 2*c)*sin(d*x + c) + 3*sqrt(2)*a*sin(d*x
+ c)^2 + 4*sqrt(2)*a*cos(d*x + c) + 2*(3*sqrt(2)*a*cos(d*x + c) + 2*sqrt(2)
*a)*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(3*d*x + 3*c) + 6*(3*sqrt(2)*a*cos(d*x
 + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + 3*(sqrt(2)*a*cos(3*d*x + 3*c)^2 + 9*s
qrt(2)*a*cos(2*d*x + 2*c)^2 + 9*sqrt(2)*a*cos(d*x + c)^2 + sqrt(2)*a*sin(3*
d*x + 3*c)^2 + 9*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 18*sqrt(2)*a*sin(2*d*x + 2*
c)*sin(d*x + c) + 9*sqrt(2)*a*sin(d*x + c)^2 + 6*sqrt(2)*a*cos(d*x + c) + 2
*(3*sqrt(2)*a*cos(2*d*x + 2*c) + 3*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(
3*d*x + 3*c) + 6*(3*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) +
6*(sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a*sin(d*x + c))*sin(3*d*x + 3*c) +
sqrt(2)*a)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6
*(sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a*sin(d*x + c) + (sqrt(2)*a*sin(2*d*
x + 2*c) + sqrt(2)*a*sin(d*x + c))*cos(3*d*x + 3*c))*sin(3*d*x + 3*c) + sqr
t(2)*a)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(s
qrt(2)*a*cos(3*d*x + 3*c)^3 + 9*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 9*sqrt(2)*a*
cos(d*x + c)^2 + 9*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 18*sqrt(2)*a*sin(2*d*x +
2*c)*sin(d*x + c) + 9*sqrt(2)*a*sin(d*x + c)^2 + 3*(2*sqrt(2)*a*cos(2*d*x +
```

```
 2*c) + 2*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(3*d*x + 3*c)^2 + (sqrt(2)
*a*cos(3*d*x + 3*c) + sqrt(2)*a)*sin(3*d*x + 3*c)^2 + 6*sqrt(2)*a*cos(d*x +
 c) + 3*(3*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 3*sqrt(2)*a*cos(d*x + c)^2 + 3*sq
rt(2)*a*sin(2*d*x + 2*c)^2 + 6*sqrt(2)*a*sin(2*d*x + 2*c)*sin(d*x + c) + 3*
sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x + c) + 2*(3*sqrt(2)*a*cos(d*
x + c) + 2*sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(3*d*x + 3*c) + 6*(3
*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + 6*(sqrt(2)*a*sin(2*
d*x + 2*c) + sqrt(2)*a*sin(d*x + c) + (sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)
*a*sin(d*x + c))*cos(3*d*x + 3*c))*sin(3*d*x + 3*c) + sqrt(2)*a)*cos(2/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*((sqrt(2)*a*sin(2*d*
x + 2*c) + sqrt(2)*a*sin(d*x + c))*cos(3*d*x + 3*c)^2 + sqrt(2)*a*sin(2*d*x
 + 2*c) + sqrt(2)*a*sin(d*x + c) + 2*(sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*
a*sin(d*x + c))*cos(3*d*x + 3*c))*sin(3*d*x + 3*c) + 6*(sqrt(2)*a*sin(3*d*x
 + 3*c)^3 + 6*(sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a*sin(d*x + c))*sin(3*d
*x + 3*c)^2 + (sqrt(2)*a*cos(3*d*x + 3*c)^2 + 9*sqrt(2)*a*cos(2*d*x + 2*c)^
2 + 9*sqrt(2)*a*cos(d*x + c)^2 + 9*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 18*sqrt(2
)*a*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sqrt(2)*a*sin(d*x + c)^2 + 6*sqrt(2)*
a*cos(d*x + c) + 2*(3*sqrt(2)*a*cos(2*d*x + 2*c) + 3*sqrt(2)*a*cos(d*x + c)
 + sqrt(2)*a)*cos(3*d*x + 3*c) + 6*(3*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*c
os(2*d*x + 2*c) + sqrt(2)*a)*sin(3*d*x + 3*c) + 3*(sqrt(2)*a*cos(3*d*x + 3*
c)^2 + 9*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 9*sqrt(2)*a*cos(d*x + c)^2 + sqrt(2
)*a*sin(3*d*x + 3*c)^2 + 9*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 18*sqrt(2)*a*sin(
2*d*x + 2*c)*sin(d*x + c) + 9*sqrt(2)*a*sin(d*x + c)^2 + 6*sqrt(2)*a*cos(d*
x + c) + 2*(3*sqrt(2)*a*cos(2*d*x + 2*c) + 3*sqrt(2)*a*cos(d*x + c) + sqrt(
2)*a)*cos(3*d*x + 3*c) + 6*(3*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x
 + 2*c) + 6*(sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a*sin(d*x + c))*sin(3*d*x
 + 3*c) + sqrt(2)*a)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(s
qrt(2)*a*sin(3*d*x + 3*c)^3 + 6*(sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a*sin
(d*x + c))*sin(3*d*x + 3*c)^2 + (sqrt(2)*a*cos(3*d*x + 3*c)^2 + 9*sqrt(2)*a
*cos(2*d*x + 2*c)^2 + 9*sqrt(2)*a*cos(d*x + c)^2 + 9*sqrt(2)*a*sin(2*d*x +
2*c)^2 + 18*sqrt(2)*a*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sqrt(2)*a*sin(d*x +
 c)^2 + 6*sqrt(2)*a*cos(d*x + c) + 2*(3*sqrt(2)*a*cos(2*d*x + 2*c) + 3*sqrt
(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(3*d*x + 3*c) + 6*(3*sqrt(2)*a*cos(d*x +
 c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(3*d*x + 3*c))*sin(2/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sqrt(a)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.45

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\left(\frac{\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}\right)}{8\,\sqrt{a}d}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(log(sin(1/2*d*x + 1/2*c) + 1)/(a*sgn(cos(1/2*d*x + 1/2*c))) -
log(-sin(1/2*d*x + 1/2*c) + 1)/(a*sgn(cos(1/2*d*x + 1/2*c))) - 2*sin(1/2*d*
x + 1/2*c)/((sin(1/2*d*x + 1/2*c)^2 - 1)*a*sgn(cos(1/2*d*x + 1/2*c))))/(sqr
t(a)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(1/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + a*cos(c + d*x))^(3/2), x)
```

3.7 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result 76
Rubi [A] (verified) 76
Mathematica [A] (verified) 77
Maple [A] (verified) 78
Fricas [B] (verification not implemented) 78
Sympy [F] 78
Maxima [B] (verification not implemented) 79
Giac [A] (verification not implemented) 130
Mupad [F(-1)] 130

Optimal result

Integrand size = 14, antiderivative size = 107

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx=\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

```
[Out] 1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+3/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c)
)^(3/2)+3/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))
/a^(5/2)/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2729, 2728, 212}

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx=\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[In] Int[(a + a*Cos[c + d*x])^(-5/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16*
Sqrt[2]*a^(5/2)*d) + Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (3*Sin
[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{3\int\frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx}{8a}\\
&= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}+\frac{3\int\frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{32a^2}\\
&= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}\\
&\quad-\frac{3\text{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,-\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{16a^2d}\\
&= \frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$
\int\frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx=\frac{24\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)+14\sin(c+dx)+3\sin(2(c+d
}{32d(a(1+\cos(c+dx)))^{5/2}}
$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(-5/2),x]
[Out] (24*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + 14*Sin[c + d*x] + 3*Sin[
2*(c + d*x)])/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.63

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(3\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{2}\,\sqrt{a}+2\sqrt{2}\,\sqrt{a\left(\right.}}{32a^{\frac{7}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

```
[In] int(1/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/a^(7/2)/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*2^(1/2)
*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*
cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x+1/2*c)^2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
2^(1/2)*a^(1/2)+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d
*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(88) = 176.

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{3\sqrt{2}\left(\cos\left(dx+c\right)^3+3\cos\left(dx+c\right)^2+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2}{\right.}}{64\left(a^3d\cos\left(dx+c\right)^3+\right.}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sq
rt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*s
in(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)
) + 4*sqrt(a*cos(d*x + c) + a)*(3*cos(d*x + c) + 7)*sin(d*x + c))/(a^3*d*co
s(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a\cos\left(c+dx\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*cos(c + d*x) + a)**(-5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84332 vs. $2(88) = 176$.

Time = 16.47 (sec) , antiderivative size = 84332, normalized size of antiderivative = 788.15

$$\int \frac{1}{(a + a\cos(c + dx))^{5/2}}\, dx = \text{Too large to display}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5
/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c)
+ (2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x + 4*
c)*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(5*d*
x + 5*c)^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*
c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d
*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c)
 + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x
+ 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*si
n(5/2*d*x + 5/2*c))*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c)
 + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x
 + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) +
 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x +
5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(
5/2*d*x + 5/2*c))*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) +
 cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x +
 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5
*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/
2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/
2*d*x + 5/2*c))*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))
)^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + co
s(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*
c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*co
s(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c
) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d
*x + 5/2*c))*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2
 - 512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*
d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) + (
2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x + 4*c)*
sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(5*d*x +
 5*c)^2 + 2560*cos(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) + 1024*(20*cos(2*d*x
 + 2*c) + 10*cos(d*x + c) + 1)*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + 1024
0*cos(3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) + 2560*sin(4*d*x + 4*c)^2*sin(5/2
```

*d*x + 5/2*c) + 10240*sin(3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) + 2560*(5*(2*
sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c
)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d
*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*s
in(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x +
 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(
8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*si
n(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*
sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x
 + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin
(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4
*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(6/
5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(
2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*si
n(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x +
 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5
/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c
)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(4/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2560*(5*(2*sin(2*d
*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5
*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/
2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*
d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*s
in(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(2/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 512*(5*cos(4*d*x + 4*
c)^2*sin(5/2*d*x + 5/2*c) + 4*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*co
s(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + 20*cos(3*d*x + 3*c)^2*sin(5/2*d*x + 5
/2*c) + 5*sin(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) + 20*sin(3*d*x + 3*c)^2*s
in(5/2*d*x + 5/2*c) + 2*((10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2
*d*x + 5/2*c) + 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c)
+ 2*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 2*(5*(2*sin(
2*d*x + 2*c) + sin(d*x + c))*sin(5/2*d*x + 5/2*c) + 10*sin(3*d*x + 3*c)*sin
(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c))*sin(4*d*x + 4*c) + 4*(5*(2*sin(2*
d*x + 2*c) + sin(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c))*sin
(3*d*x + 3*c) + (4*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 20*cos(2*d*x + 2
*c)^2 + 5*cos(d*x + c)^2 + 20*sin(2*d*x + 2*c)^2 + 20*sin(2*d*x + 2*c)*sin(
d*x + c) + 5*sin(d*x + c)^2 + 2*cos(d*x + c))*sin(5/2*d*x + 5/2*c))*cos(5*d
*x + 5*c) + 512*((20*cos(2*d*x + 2*c) + 10*cos(d*x + c) + 1)*sin(5/2*d*x +
5/2*c) + 20*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c) + 512*(
2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) - 12*(10*(sin(4*d*x
 + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x
 + 5*c)^2 + sin(5*d*x + 5*c)^3 + (2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*
c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x
 + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c)
+ 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5

```
*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*
x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 +
 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*
c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*
d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*
x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(5*d*x
+ 5*c) + 5*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*
c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos
(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c)
+ 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos
(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x
+ 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c)
+ 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c)
+ sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*
x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c)
+ sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x +
 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*
x + c) + 1)*sin(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
10*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*c
os(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x +
 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos
(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x +
 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) +
 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(
3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*
d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*
sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*
x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2
+ 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) +
 1)*sin(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*(2*(5
*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x +
 c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) +
10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x +
 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) +
100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos
(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x +
3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*
c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*
x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*
sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*si
n(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(
4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*(2*(5*cos(4*d*
x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*
cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*
d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 +
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 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3
*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x +
2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*
sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50
*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c)
+ 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x
 + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x +
 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(2/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*cos(9/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 4*(128*cos(5*d*x + 5*c)^2*sin(5/2*d*x
+ 5/2*c) - 3200*cos(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) - 12800*(2*cos(2*d*
x + 2*c) + cos(d*x + c))*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) - 12800*cos(
3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) - 128*sin(5*d*x + 5*c)^2*sin(5/2*d*x +
5/2*c) - 3200*sin(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) - 12800*sin(3*d*x + 3
*c)^2*sin(5/2*d*x + 5/2*c) - 256*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos
(5/2*d*x + 5/2*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*
x + 5/2*c)*sin(3*d*x + 3*c) - sin(5/2*d*x + 5/2*c))*cos(5*d*x + 5*c) - 6400
*((2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x +
3*c)*sin(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c) - 1280*(2*sin(2*d*x + 2*c) + si
n(d*x + c))*cos(5/2*d*x + 5/2*c) - 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x +
c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/
2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c)
- (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d
*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) -
10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(6/5*arctan2(sin(5/2*d*x + 5/2
*c), cos(5/2*d*x + 5/2*c))) - 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*c
os(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x
 + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10
*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x +
5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*co
s(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c),
cos(5/2*d*x + 5/2*c))) - 1280*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/
2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/
2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(
2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*
sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d
*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))) - 256*(cos(5*d*x + 5*c)*cos(5/2*d*x + 5/2*c) + 5*(2*sin(2
*d*x + 2*c) + sin(d*x + c))*sin(5/2*d*x + 5/2*c) + 5*sin(4*d*x + 4*c)*sin(5
/2*d*x + 5/2*c) + 10*sin(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + cos(5/2*d*x +
5/2*c))*sin(5*d*x + 5*c) - 1280*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(
5/2*d*x + 5/2*c) + 10*sin(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + cos(5/2*d*x +
 5/2*c))*sin(4*d*x + 4*c) - 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin
(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c))*sin(3*d*x + 3*c) - 128*(100*cos(2
*d*x + 2*c)^2 + 100*cos(2*d*x + 2*c)*cos(d*x + c) + 25*cos(d*x + c)^2 + 100
```

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*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2
 - 1)*sin(5/2*d*x + 5/2*c) - 70*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c
) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x
+ 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) +
 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*
cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x
 + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 +
10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x +
c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(
2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100
*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c
)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d
*x + c)^2 + 10*cos(d*x + c) + 1)*sin(7/5*arctan2(sin(5/2*d*x + 5/2*c), cos(
5/2*d*x + 5/2*c))) + 70*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*c
os(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2
 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(
4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x
+ c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) +
1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(
4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(
5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x +
2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(
2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 10
0*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^
2 + 10*cos(d*x + c) + 1)*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
+ 5/2*c))) + 15*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x
 + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(1
0*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x +
4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1
)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2
*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x +
4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x +
5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + s
in(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x +
2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*
d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*c
os(d*x + c) + 1)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
)))*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 56*(10*(
sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*
sin(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^3 + (2*(5*cos(4*d*x + 4*c) + 10*cos(3
*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) +
cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(
d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x +
 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*
(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x
```

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 + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4
*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c
))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100
*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*s
in(5*d*x + 5*c) + 10*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(
2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 +
10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d
*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c
) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*
cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d
*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d
*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c
) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d
*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*s
in(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 +
 10*cos(d*x + c) + 1)*sin(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5
/2*c))) + 10*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x +
2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*c
os(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c
) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*c
os(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*
x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c
) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c
) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(
d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c
) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x
 + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(
d*x + c) + 1)*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))
+ 5*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*
cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x
+ 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*co
s(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x
+ 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c)
+ 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin
(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5
*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))
*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d
*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2
 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c)
+ 1)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*cos(7/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 8*(128*cos(5*d*x + 5
*c)^2*sin(5/2*d*x + 5/2*c) - 3200*cos(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) -
 12800*(2*cos(2*d*x + 2*c) + cos(d*x + c))*cos(3*d*x + 3*c)*sin(5/2*d*x + 5
/2*c) - 12800*cos(3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) - 128*sin(5*d*x + 5*c
)^2*sin(5/2*d*x + 5/2*c) - 3200*sin(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) - 1
```

```
2800*sin(3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) - 256*(5*(2*sin(2*d*x + 2*c) +
 sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*
c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - sin(5/2*d*x + 5/2*c))*cos(5
*d*x + 5*c) - 6400*((2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c
) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c) - 1280*(2*sin
(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) - 2560*(5*(2*sin(2*d*x +
 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x
 + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)
*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x
+ 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5
/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(4/5*arctan2
(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 1280*(5*(2*sin(2*d*x + 2*c)
 + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*
c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(
3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2
*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*
x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(2/5*arctan2(sin(
5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 256*(cos(5*d*x + 5*c)*cos(5/2*d*
x + 5/2*c) + 5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5/2*d*x + 5/2*c) + 5
*sin(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) + 10*sin(3*d*x + 3*c)*sin(5/2*d*x +
5/2*c) + cos(5/2*d*x + 5/2*c))*sin(5*d*x + 5*c) - 1280*(5*(2*sin(2*d*x + 2*
c) + sin(d*x + c))*sin(5/2*d*x + 5/2*c) + 10*sin(3*d*x + 3*c)*sin(5/2*d*x +
 5/2*c) + cos(5/2*d*x + 5/2*c))*sin(4*d*x + 4*c) - 2560*(5*(2*sin(2*d*x + 2
*c) + sin(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c))*sin(3*d*x
+ 3*c) - 128*(100*cos(2*d*x + 2*c)^2 + 100*cos(2*d*x + 2*c)*cos(d*x + c) +
25*cos(d*x + c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x +
 c) + 25*sin(d*x + c)^2 - 1)*sin(5/2*d*x + 5/2*c) + 70*(2*(5*cos(4*d*x + 4*
c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*
d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x +
2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(1
0*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x +
 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2
 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*
d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*si
n(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*s
in(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c
) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*
sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(3/5*arctan2(sin
(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 15*(2*(5*cos(4*d*x + 4*c) + 10*
cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*
c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5
*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*
d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2
+ 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*co
s(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*
```

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c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x
+ 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x
 + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*
sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x
+ c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(1/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c))))*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), co
s(5/2*d*x + 5/2*c))) - 8*(128*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) - 320
0*cos(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) - 12800*(2*cos(2*d*x + 2*c) + cos
(d*x + c))*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) - 12800*cos(3*d*x + 3*c)^2
*sin(5/2*d*x + 5/2*c) - 128*sin(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) - 3200*
sin(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) - 12800*sin(3*d*x + 3*c)^2*sin(5/2*
d*x + 5/2*c) - 256*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2
*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin
(3*d*x + 3*c) - sin(5/2*d*x + 5/2*c))*cos(5*d*x + 5*c) - 6400*((2*cos(2*d*x
 + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d
*x + 5/2*c))*cos(4*d*x + 4*c) - 1280*(2*sin(2*d*x + 2*c) + sin(d*x + c))*co
s(5/2*d*x + 5/2*c) - 1280*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*
x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)
*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*
x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(
5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x +
 3*c)*sin(5/2*d*x + 5/2*c))*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c))) - 256*(cos(5*d*x + 5*c)*cos(5/2*d*x + 5/2*c) + 5*(2*sin(2*d*x
 + 2*c) + sin(d*x + c))*sin(5/2*d*x + 5/2*c) + 5*sin(4*d*x + 4*c)*sin(5/2*d
*x + 5/2*c) + 10*sin(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*
c))*sin(5*d*x + 5*c) - 1280*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5/2*
d*x + 5/2*c) + 10*sin(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2
*c))*sin(4*d*x + 4*c) - 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5/2
*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c))*sin(3*d*x + 3*c) - 128*(100*cos(2*d*x
 + 2*c)^2 + 100*cos(2*d*x + 2*c)*cos(d*x + c) + 25*cos(d*x + c)^2 + 100*sin
(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 - 1
)*sin(5/2*d*x + 5/2*c) + 70*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) +
10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*
c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*
cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(
d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c
) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(
sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*
sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*
x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*
sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2
+ 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x +
 c)^2 + 10*cos(d*x + c) + 1)*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c))) + 15*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2
*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 1
```

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0*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*
x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c)
 + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*c
os(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*
x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*
x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c)
 + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*
x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*si
n(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 +
10*cos(d*x + c) + 1)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/
2*c))))*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 56*(
10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x +
c))*sin(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^3 + (2*(5*cos(4*d*x + 4*c) + 10*c
os(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c
) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*
cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d
*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 +
 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos
(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*s
in(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x
 + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 +
 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) +
1)*sin(5*d*x + 5*c) + 5*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*c
os(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2
 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(
4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x
+ c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) +
1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(
4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(
5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x +
2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(
2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 10
0*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^
2 + 10*cos(d*x + c) + 1)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
+ 5/2*c))))*cos(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) -
4*(128*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) - 3200*cos(4*d*x + 4*c)^2*si
n(5/2*d*x + 5/2*c) - 12800*(2*cos(2*d*x + 2*c) + cos(d*x + c))*cos(3*d*x +
3*c)*sin(5/2*d*x + 5/2*c) - 12800*cos(3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) -
 128*sin(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) - 3200*sin(4*d*x + 4*c)^2*sin(
5/2*d*x + 5/2*c) - 12800*sin(3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) - 256*(5*(
2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 5*cos(5/2*d*x + 5
/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - sin(5/2
*d*x + 5/2*c))*cos(5*d*x + 5*c) - 6400*((2*cos(2*d*x + 2*c) + cos(d*x + c))
*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(4*d*x
+ 4*c) - 1280*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) - 25
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6*(cos(5*d*x + 5*c)*cos(5/2*d*x + 5/2*c) + 5*(2*sin(2*d*x + 2*c) + sin(d*x
+ c))*sin(5/2*d*x + 5/2*c) + 5*sin(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) + 10*s
in(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c))*sin(5*d*x + 5*
c) - 1280*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5/2*d*x + 5/2*c) + 10*
sin(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c))*sin(4*d*x + 4
*c) - 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5/2*d*x + 5/2*c) + co
s(5/2*d*x + 5/2*c))*sin(3*d*x + 3*c) - 128*(100*cos(2*d*x + 2*c)^2 + 100*co
s(2*d*x + 2*c)*cos(d*x + c) + 25*cos(d*x + c)^2 + 100*sin(2*d*x + 2*c)^2 +
100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 - 1)*sin(5/2*d*x + 5/
2*c) + 15*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c
) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(
3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) +
 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(
3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x +
 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) +
 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) +
 sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x
 + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) +
 sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x +
2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x
 + c) + 1)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*co
s(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 12*(10*(sin(4*
d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*
d*x + 5*c)^2 + sin(5*d*x + 5*c)^3 + (2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x +
 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*
d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x +
c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c)
+ 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos
(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^
2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x +
 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin
(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2
*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(5*d
*x + 5*c))*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 3
*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos
(d*x + c) + 2)*cos(5*d*x + 5*c)^3 + cos(5*d*x + 5*c)^4 + 10*(sin(4*d*x + 4*
c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*
c)^3 + sin(5*d*x + 5*c)^4 + (10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c)
+ 5*cos(d*x + c) + 3)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos
(2*d*x + 2*c) + 5*cos(d*x + c) + 3)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)
^2 + 20*(5*cos(d*x + c) + 3)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25
*cos(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c
))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin
(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)
^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 30*cos(d*x + c
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) + 6)*cos(5*d*x + 5*c)^2 + 25*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c)
 + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x +
 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) +
1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*c
os(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x
+ c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 1
0*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c
))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2
*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*
(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)
^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*
x + c)^2 + 10*cos(d*x + c) + 1)*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c)))^2 + 100*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10
*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)
^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*co
s(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*
x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c)
+ 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(si
n(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*si
n(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x
+ 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*si
n(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 +
100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c
)^2 + 10*cos(d*x + c) + 1)*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*
x + 5/2*c)))^2 + 100*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(
2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 +
10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d
*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c
) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*
cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d
*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d
*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c
) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d
*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*s
in(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 +
 10*cos(d*x + c) + 1)*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5
/2*c)))^2 + 25*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x
+ 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10
*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4
*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)
*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*
d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4
*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5
*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + si
n(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2
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*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d
*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*co
s(d*x + c) + 1)*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))
)^2 + (2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) +
5*cos(d*x + c) + 2)*cos(5*d*x + 5*c) + 2*cos(5*d*x + 5*c)^2 + 10*(10*cos(3*
d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 2
5*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*
d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2
*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c) +
 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^
2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*
x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 2
5*sin(d*x + c)^2 + 10*cos(d*x + c) + 2)*sin(5*d*x + 5*c)^2 + 25*(2*(5*cos(4
*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) +
1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos
(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^
2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*co
s(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x
 + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) +
 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 +
 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*
c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*
d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*
x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(8/5*ar
ctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 100*(2*(5*cos(4*d*x
+ 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*co
s(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*
x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 2
0*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d
*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*
c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*si
n(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(
2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) +
25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x +
 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2
*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(6/5*arctan2
(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 100*(2*(5*cos(4*d*x + 4*c
) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d
*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2
*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10
*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x +
3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2
+ 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d
*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin
(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*si
```

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n(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c)
 + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*s
in(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(4/5*arctan2(sin(
5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 25*(2*(5*cos(4*d*x + 4*c) + 10
*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5
*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) +
5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2
*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2
 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*c
os(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2
*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x
 + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*
x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100
*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x
 + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(2/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*(5*(20*cos(3*d*x + 3*c) + 20*cos(2
*d*x + 2*c) + 10*cos(d*x + c) + 3)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2
 + 10*(20*cos(2*d*x + 2*c) + 10*cos(d*x + c) + 3)*cos(3*d*x + 3*c) + 100*co
s(3*d*x + 3*c)^2 + 10*(10*cos(d*x + c) + 3)*cos(2*d*x + 2*c) + 100*cos(2*d*
x + 2*c)^2 + 25*cos(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c
) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d
*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*s
in(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 +
 15*cos(d*x + c) + 2)*cos(5*d*x + 5*c) + 10*(10*cos(3*d*x + 3*c) + 10*cos(2
*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2
+ 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(
3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x +
 2*c)^2 + 25*cos(d*x + c)^2 + 10*((10*cos(4*d*x + 4*c) + 20*cos(3*d*x + 3*c
) + 20*cos(2*d*x + 2*c) + 10*cos(d*x + c) + 3)*cos(5*d*x + 5*c)^2 + cos(5*d
*x + 5*c)^3 + (cos(5*d*x + 5*c) + 1)*sin(5*d*x + 5*c)^2 + (10*(10*cos(3*d*x
 + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 2)*cos(4*d*x + 4*c) + 25*c
os(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 2)*cos(3*d*x
 + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 2)*cos(2*d*x + 2*c)
 + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*
sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 +
 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x +
 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*s
in(d*x + c)^2 + 20*cos(d*x + c) + 3)*cos(5*d*x + 5*c) + 10*(10*cos(3*d*x +
3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(
4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x +
3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) +
100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(2*(5*cos(4*d*x + 4*c) + 10
*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5
*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) +
5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2
```

```
*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2
 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*c
os(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2
*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x
 + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*
x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100
*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x
 + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(6/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d
*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + co
s(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*
x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2
*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5
*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x +
 c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + si
n(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c)
+ 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)
^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d
*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) +
25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(4/5*arctan2(sin(5/2*d*x + 5/2*
c), cos(5/2*d*x + 5/2*c))) + 5*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c)
 + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x +
 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) +
1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*c
os(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x
+ c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 1
0*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c
))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2
*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*
(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)
^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*
x + c)^2 + 10*cos(d*x + c) + 1)*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))) + 10*((sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*
x + 2*c) + sin(d*x + c))*cos(5*d*x + 5*c) + sin(4*d*x + 4*c) + 2*sin(3*d*x
+ 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sin(3*
d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4
*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) +
100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(
d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(8/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*((10*cos(4*d*x + 4*c) + 20*cos(3
*d*x + 3*c) + 20*cos(2*d*x + 2*c) + 10*cos(d*x + c) + 3)*cos(5*d*x + 5*c)^2
 + cos(5*d*x + 5*c)^3 + (cos(5*d*x + 5*c) + 1)*sin(5*d*x + 5*c)^2 + (10*(10
*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 2)*cos(4*d*x + 4
*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 2)
*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 2)*cos(2*
```

```
d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 50*(2*sin(3*d*x +
 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x
+ 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*s
in(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x +
 c) + 25*sin(d*x + c)^2 + 20*cos(d*x + c) + 3)*cos(5*d*x + 5*c) + 10*(10*co
s(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c)
 + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*co
s(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x
 + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(2*(5*cos(4*d*x +
 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos
(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x
 + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20
*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*
x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c
)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin
(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2
*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 2
5*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x +
3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*
c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(4/5*arctan2(
sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*(2*(5*cos(4*d*x + 4*c) + 1
0*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x +
5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) +
 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(
2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^
2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*
cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x +
2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*
x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d
*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 10
0*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*
x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(2/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*((sin(4*d*x + 4*c) + 2*sin(3*d*x +
 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5*d*x + 5*c) + sin(4*d*x + 4
*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5
*c) + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x
 + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*s
in(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin
(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(6
/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*((10*cos(4*d*x
 + 4*c) + 20*cos(3*d*x + 3*c) + 20*cos(2*d*x + 2*c) + 10*cos(d*x + c) + 3)*
cos(5*d*x + 5*c)^2 + cos(5*d*x + 5*c)^3 + (cos(5*d*x + 5*c) + 1)*sin(5*d*x
+ 5*c)^2 + (10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c)
+ 2)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5
*cos(d*x + c) + 2)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*
```

```
x + c) + 2)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 +
 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*
c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*
d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*
x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 20*cos(d*x + c) + 3)*cos(5*d*x
+ 5*c) + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1
)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*co
s(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x +
 c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 5*
(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(
d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*
c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*
d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*
c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 10
0*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d
*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x
 + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin
(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x +
 c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 1
00*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)
*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*((sin(4*
d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5*
d*x + 5*c) + sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + s
in(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c)
 + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*
x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*si
n(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 +
10*cos(d*x + c) + 1)*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/
2*c))) + 10*((10*cos(4*d*x + 4*c) + 20*cos(3*d*x + 3*c) + 20*cos(2*d*x + 2*
c) + 10*cos(d*x + c) + 3)*cos(5*d*x + 5*c)^2 + cos(5*d*x + 5*c)^3 + (cos(5*
d*x + 5*c) + 1)*sin(5*d*x + 5*c)^2 + (10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*
x + 2*c) + 5*cos(d*x + c) + 2)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 2
0*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 2)*cos(3*d*x + 3*c) + 100*cos(3*d
*x + 3*c)^2 + 20*(5*cos(d*x + c) + 2)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*
c)^2 + 25*cos(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + si
n(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2
*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d
*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 20*co
s(d*x + c) + 3)*cos(5*d*x + 5*c) + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x +
 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(
10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x
+ 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^
2 + 25*cos(d*x + c)^2 + 10*((sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(
2*d*x + 2*c) + sin(d*x + c))*cos(5*d*x + 5*c) + sin(4*d*x + 4*c) + 2*sin(3*
d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*si
```

```
n(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*s
in(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c
) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*
sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(2/5*arctan2(sin
(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*((sin(4*d*x + 4*c) + 2*sin(3
*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5*d*x + 5*c)^2 + 2*(si
n(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*co
s(5*d*x + 5*c) + sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c)
 + sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x +
2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(
2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 10
0*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^
2 + 10*(10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + si
n(d*x + c))*sin(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^3 + (2*(5*cos(4*d*x + 4*c
) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d
*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2
*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10
*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x +
3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2
+ 25*cos(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x
 + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) +
 sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x +
2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x
 + c) + 1)*sin(5*d*x + 5*c) + 10*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*
c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x
 + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c)
+ 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5
*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*
x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 +
 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x +
 c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin
(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 10
0*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*
c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(
d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos
(5/2*d*x + 5/2*c))) + 10*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*
cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^
2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos
(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x
 + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) +
 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin
(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin
(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x +
 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin
(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 1
```

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00*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)
^2 + 10*cos(d*x + c) + 1)*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
 + 5/2*c))) + 5*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x
 + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(1
0*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x +
4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1
)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2
*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x +
4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x +
5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + s
in(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x +
2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*
d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*c
os(d*x + c) + 1)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
)))*sin(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*(10*(
sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*
sin(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^3 + (2*(5*cos(4*d*x + 4*c) + 10*cos(3
*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) +
cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(
d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x +
 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*
(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x
 + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4
*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c
))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100
*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*s
in(5*d*x + 5*c) + 10*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(
2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 +
10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d
*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c
) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*
cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d
*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d
*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c
) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d
*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*s
in(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 +
 10*cos(d*x + c) + 1)*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5
/2*c))) + 5*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2
*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*co
s(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c)
 + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*co
s(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x
 + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c)
 + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c)
```

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 + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d
*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c)
 + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x
+ 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d
*x + c) + 1)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*
sin(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*(10*(sin(
4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(
5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^3 + (2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x
 + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(
5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x
+ c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c
) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*c
os(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c
)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x
 + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*s
in(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin
(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(5
*d*x + 5*c) + 5*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x
 + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(1
0*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x +
4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1
)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2
*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x +
4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x +
5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + s
in(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x +
2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*
d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*c
os(d*x + c) + 1)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
)))*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*(10*(
sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*
sin(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^3 + (2*(5*cos(4*d*x + 4*c) + 10*cos(3
*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) +
cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(
d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x +
 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*
(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x
 + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4
*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c
))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100
*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*s
in(5*d*x + 5*c))*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
)) + 10*cos(d*x + c) + 1)*log(cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
```

```
1) - 3*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) +
 5*cos(d*x + c) + 2)*cos(5*d*x + 5*c)^3 + cos(5*d*x + 5*c)^4 + 10*(sin(4*d*
x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*
x + 5*c)^3 + sin(5*d*x + 5*c)^4 + (10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x +
 2*c) + 5*cos(d*x + c) + 3)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(
10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 3)*cos(3*d*x + 3*c) + 100*cos(3*d*x
+ 3*c)^2 + 20*(5*cos(d*x + c) + 3)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^
2 + 25*cos(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d
*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c)
 + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x
+ 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 30*cos(d
*x + c) + 6)*cos(5*d*x + 5*c)^2 + 25*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x
+ 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5
*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x +
 c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c)
 + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*co
s(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)
^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d
*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2
*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2
+ 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x
+ 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*
sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c),
 cos(5/2*d*x + 5/2*c)))^2 + 100*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c
) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x
+ 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) +
 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*
cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x
 + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 +
10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x +
c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(
2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100
*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c
)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d
*x + c)^2 + 10*cos(d*x + c) + 1)*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(
5/2*d*x + 5/2*c)))^2 + 100*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 1
0*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c
)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*c
os(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d
*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c)
 + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(s
in(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*s
in(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x
 + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*s
in(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 +
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100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 25*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + (2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 2)*cos(5*d*x + 5*c) + 2*cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 2)*sin(5*d*x + 5*c)^2 + 25*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 100*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3

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*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d
*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(6/5*a
rctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 100*(2*(5*cos(4*d*x
 + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*c
os(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d
*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 +
20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*
d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2
*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*s
in(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*
(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) +
 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x
+ 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x +
2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 25*(2*(5*cos(4*d*x + 4*c
) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d
*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2
*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10
*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x +
3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2
+ 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d
*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin
(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*si
n(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c)
 + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*s
in(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(2/5*arctan2(sin(
5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*(5*(20*cos(3*d*x + 3*c) + 20
*cos(2*d*x + 2*c) + 10*cos(d*x + c) + 3)*cos(4*d*x + 4*c) + 25*cos(4*d*x +
4*c)^2 + 10*(20*cos(2*d*x + 2*c) + 10*cos(d*x + c) + 3)*cos(3*d*x + 3*c) +
100*cos(3*d*x + 3*c)^2 + 10*(10*cos(d*x + c) + 3)*cos(2*d*x + 2*c) + 100*co
s(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x
 + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*s
in(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 +
 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x +
c)^2 + 15*cos(d*x + c) + 2)*cos(5*d*x + 5*c) + 10*(10*cos(3*d*x + 3*c) + 10
*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4
*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 10
0*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2
*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*((10*cos(4*d*x + 4*c) + 20*cos(3*d*x
 + 3*c) + 20*cos(2*d*x + 2*c) + 10*cos(d*x + c) + 3)*cos(5*d*x + 5*c)^2 + c
os(5*d*x + 5*c)^3 + (cos(5*d*x + 5*c) + 1)*sin(5*d*x + 5*c)^2 + (10*(10*cos
(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 2)*cos(4*d*x + 4*c)
+ 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 2)*cos
(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 2)*cos(2*d*x
+ 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c
```

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) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*
c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3
*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c)
+ 25*sin(d*x + c)^2 + 20*cos(d*x + c) + 3)*cos(5*d*x + 5*c) + 10*(10*cos(3*
d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 2
5*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*
d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2
*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(2*(5*cos(4*d*x + 4*c
) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d
*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2
*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10
*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x +
3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2
+ 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d
*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin
(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*si
n(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c)
 + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*s
in(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(6/5*arctan2(sin(
5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*(2*(5*cos(4*d*x + 4*c) + 10*c
os(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c
) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*
cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d
*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 +
 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos
(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c
) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x +
 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x
+ 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*s
in(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x +
 c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(4/5*arctan2(sin(5/2*d*x
+ 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x
+ 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5
*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x +
 c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c)
 + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*co
s(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)
^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d
*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2
*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2
+ 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x
+ 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*
sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c),
 cos(5/2*d*x + 5/2*c))) + 10*((sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*si
n(2*d*x + 2*c) + sin(d*x + c))*cos(5*d*x + 5*c) + sin(4*d*x + 4*c) + 2*sin(
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3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*
sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25
*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3
*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c
)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(8/5*arctan2(s
in(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*((10*cos(4*d*x + 4*c) + 20
*cos(3*d*x + 3*c) + 20*cos(2*d*x + 2*c) + 10*cos(d*x + c) + 3)*cos(5*d*x +
5*c)^2 + cos(5*d*x + 5*c)^3 + (cos(5*d*x + 5*c) + 1)*sin(5*d*x + 5*c)^2 + (
10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 2)*cos(4*d
*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c
) + 2)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 2)*
cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 50*(2*sin(3
*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(
4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) +
100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin
(d*x + c) + 25*sin(d*x + c)^2 + 20*cos(d*x + c) + 3)*cos(5*d*x + 5*c) + 10*
(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x
+ 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) +
1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos
(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(2*(5*cos(4
*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) +
1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos
(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^
2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*co
s(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x
+ 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) +
2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 +
50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*
c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*
d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*
x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(4/5*ar
ctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*(2*(5*cos(4*d*x + 4*
c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*
d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x +
2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(1
0*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x +
3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2
+ 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*
d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*si
n(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*s
in(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c
) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*
sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(2/5*arctan2(sin
(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*((sin(4*d*x + 4*c) + 2*sin(3
*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5*d*x + 5*c) + sin(4*d

```
*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d
*x + 5*c) + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin
(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x +
 c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 1
00*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)
*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*((10*cos
(4*d*x + 4*c) + 20*cos(3*d*x + 3*c) + 20*cos(2*d*x + 2*c) + 10*cos(d*x + c)
 + 3)*cos(5*d*x + 5*c)^2 + cos(5*d*x + 5*c)^3 + (cos(5*d*x + 5*c) + 1)*sin(
5*d*x + 5*c)^2 + (10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x
 + c) + 2)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*
c) + 5*cos(d*x + c) + 2)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*
cos(d*x + c) + 2)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x +
c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*
x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*
sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*si
n(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 20*cos(d*x + c) + 3)*cos(
5*d*x + 5*c) + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x +
c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c)
+ 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos
(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^
2 + 5*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) +
5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*
x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*
cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*
x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c
) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*s
in(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin
(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c
))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin
(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)
^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c
) + 1)*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*((
sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*
cos(5*d*x + 5*c) + sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*
c) + sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x
+ 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*si
n(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 +
100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c
)^2 + 10*cos(d*x + c) + 1)*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*
x + 5/2*c))) + 10*((10*cos(4*d*x + 4*c) + 20*cos(3*d*x + 3*c) + 20*cos(2*d*
x + 2*c) + 10*cos(d*x + c) + 3)*cos(5*d*x + 5*c)^2 + cos(5*d*x + 5*c)^3 + (
cos(5*d*x + 5*c) + 1)*sin(5*d*x + 5*c)^2 + (10*(10*cos(3*d*x + 3*c) + 10*co
s(2*d*x + 2*c) + 5*cos(d*x + c) + 2)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)
^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 2)*cos(3*d*x + 3*c) + 100*c
os(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 2)*cos(2*d*x + 2*c) + 100*cos(2*d*
```

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x + 2*c)^2 + 25*cos(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c
) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d
*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*s
in(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 +
 20*cos(d*x + c) + 3)*cos(5*d*x + 5*c) + 10*(10*cos(3*d*x + 3*c) + 10*cos(2
*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2
+ 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(
3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x +
 2*c)^2 + 25*cos(d*x + c)^2 + 10*((sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) +
2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5*d*x + 5*c) + sin(4*d*x + 4*c) + 2*
sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + 50
*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c)
+ 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x
 + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x +
 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(2/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*((sin(4*d*x + 4*c) + 2
*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5*d*x + 5*c)^2 +
 2*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x +
c))*cos(5*d*x + 5*c) + sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x
+ 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*
d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(
2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^
2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x
 + c)^2 + 10*(10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c
) + sin(d*x + c))*sin(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^3 + (2*(5*cos(4*d*x
 + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*c
os(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d
*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 +
20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*
d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2
*c)^2 + 25*cos(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + s
in(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x +
2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*
d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*c
os(d*x + c) + 1)*sin(5*d*x + 5*c) + 10*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*
x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos
(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x
 + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*
c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*
cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x +
c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin
(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) +
 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^
2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*
x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 2
```

```
5*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(6/5*arctan2(sin(5/2*d*x + 5/2*c
), cos(5/2*d*x + 5/2*c))) + 10*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c)
 + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x +
 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) +
1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*c
os(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x
+ c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 1
0*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c
))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2
*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*
(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)
^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*
x + c)^2 + 10*cos(d*x + c) + 1)*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))) + 5*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos
(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 +
 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*
d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x +
c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)
*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*
d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*
d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*
c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*
d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*
sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2
+ 10*cos(d*x + c) + 1)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
5/2*c))))*sin(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20
*(10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x
+ c))*sin(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^3 + (2*(5*cos(4*d*x + 4*c) + 10
*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5
*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) +
5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2
*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2
 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*c
os(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))
*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d
*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2
 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c)
+ 1)*sin(5*d*x + 5*c) + 10*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 1
0*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c
)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*c
os(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d
*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c)
 + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(s
in(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*s
in(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x
```

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 + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*s
in(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 +
 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x +
c)^2 + 10*cos(d*x + c) + 1)*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c))) + 5*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d
*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*
(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x
+ 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) +
 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos
(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x
+ 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x
+ 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) +
 sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x
+ 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(
2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10
*cos(d*x + c) + 1)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*
c))))*sin(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*(10
*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c)
)*sin(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^3 + (2*(5*cos(4*d*x + 4*c) + 10*cos
(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c)
+ cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*co
s(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x
 + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 2
0*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d
*x + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin
(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x +
 c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 1
00*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)
*sin(5*d*x + 5*c) + 5*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos
(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 +
 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*
d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x +
c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)
*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*
d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*
d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*
c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*
d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*
sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2
+ 10*cos(d*x + c) + 1)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
5/2*c))))*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10
*(10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x
+ c))*sin(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^3 + (2*(5*cos(4*d*x + 4*c) + 10
*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5
*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) +
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5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2
*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2
 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*c
os(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))
*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d
*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2
 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c)
+ 1)*sin(5*d*x + 5*c))*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
5/2*c))) + 10*cos(d*x + c) + 1)*log(cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), c
os(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
 + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
))) + 1) - 512*(5*cos(4*d*x + 4*c)^2*cos(5/2*d*x + 5/2*c) + 4*(10*cos(2*d*x
 + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c)*cos(5/2*d*x + 5/2*c) + 20*co
s(3*d*x + 3*c)^2*cos(5/2*d*x + 5/2*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x +
4*c)^2 + 20*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c)^2 + 2*((2*cos(2*d*x + 2*c
) + cos(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(4*d*x + 4*c)*cos(5/2*d*x + 5/2
*c) + 2*cos(3*d*x + 3*c)*cos(5/2*d*x + 5/2*c) - (2*sin(2*d*x + 2*c) + sin(d
*x + c))*sin(5/2*d*x + 5/2*c) - sin(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 2*s
in(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(5*d*x + 5*c) + 2*((10*cos(2*d*x +
 2*c) + 5*cos(d*x + c) + 1)*cos(5/2*d*x + 5/2*c) + 10*cos(3*d*x + 3*c)*cos(
5/2*d*x + 5/2*c))*cos(4*d*x + 4*c) + (4*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*
c) + 20*cos(2*d*x + 2*c)^2 + 5*cos(d*x + c)^2 + 20*sin(2*d*x + 2*c)^2 + 20*
sin(2*d*x + 2*c)*sin(d*x + c) + 5*sin(d*x + c)^2 + 2*cos(d*x + c))*cos(5/2*
d*x + 5/2*c) + 2*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c
) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - sin(5/2*d*x + 5/2*c))*sin(4*
d*x + 4*c) + 4*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c)
- sin(5/2*d*x + 5/2*c))*sin(3*d*x + 3*c) - 2*(2*sin(2*d*x + 2*c) + sin(d*x
+ c))*sin(5/2*d*x + 5/2*c))*sin(5*d*x + 5*c) + 512*(10*(2*sin(2*d*x + 2*c)
+ sin(d*x + c))*sin(5/2*d*x + 5/2*c) + 20*sin(3*d*x + 3*c)*sin(5/2*d*x + 5/
2*c) + cos(5/2*d*x + 5/2*c))*sin(4*d*x + 4*c) + 1024*(10*(2*sin(2*d*x + 2*c
) + sin(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c))*sin(3*d*x +
3*c) + 512*(2*(10*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 20*cos(2*d*x + 2*c)^
2 + 5*cos(d*x + c)^2 + 20*sin(2*d*x + 2*c)^2 + 20*sin(2*d*x + 2*c)*sin(d*x
+ c) + 5*sin(d*x + c)^2 + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + 12*((10*cos(
4*d*x + 4*c) + 20*cos(3*d*x + 3*c) + 20*cos(2*d*x + 2*c) + 10*cos(d*x + c)
+ 3)*cos(5*d*x + 5*c)^2 + cos(5*d*x + 5*c)^3 + (cos(5*d*x + 5*c) + 1)*sin(5
*d*x + 5*c)^2 + (10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x
+ c) + 2)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c
) + 5*cos(d*x + c) + 2)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*c
os(d*x + c) + 2)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c
)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x
 + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*s
in(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin
(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 20*cos(d*x + c) + 3)*cos(5
*d*x + 5*c) + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c
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) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) +
 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(
d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2
 + 5*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5
*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x
 + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*c
os(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x
 + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c)
 + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*si
n(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(
5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c)
)*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(
d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^
2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c)
 + 1)*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*(2*
(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x
 + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c)
+ 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x
 + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c)
+ 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*c
os(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x
+ 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x +
5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*
d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c)
)*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*
sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*co
s(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*(2*(5*cos(4
*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) +
1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos
(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^
2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*co
s(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x
 + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) +
 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 +
 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*
c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*
d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*
x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(4/5*ar
ctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*(2*(5*cos(4*d*x + 4*
c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*
d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x +
2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(1
0*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x +
 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2
 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*
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d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*si
n(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*s
in(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c
) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*
sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(2/5*arctan2(sin
(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*((sin(4*d*x + 4*c) + 2*sin(3
*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5*d*x + 5*c) + sin(4*d
*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d
*x + 5*c) + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin
(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x +
 c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 1
00*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)
*sin(9/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 4*(128*cos(
5*d*x + 5*c)^2*cos(5/2*d*x + 5/2*c) + 3200*cos(4*d*x + 4*c)^2*cos(5/2*d*x +
 5/2*c) + 2560*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c)*
cos(5/2*d*x + 5/2*c) + 12800*cos(3*d*x + 3*c)^2*cos(5/2*d*x + 5/2*c) - 128*
cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c)^2 + 3200*cos(5/2*d*x + 5/2*c)*sin(4*d
*x + 4*c)^2 + 12800*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c
)*sin(3*d*x + 3*c) + 12800*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c)^2 + 256*((
10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5/2*d*x + 5/2*c) + 5*cos(4*d*
x + 4*c)*cos(5/2*d*x + 5/2*c) + 10*cos(3*d*x + 3*c)*cos(5/2*d*x + 5/2*c))*c
os(5*d*x + 5*c) + 1280*((10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5/2*
d*x + 5/2*c) + 10*cos(3*d*x + 3*c)*cos(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c) +
 128*(20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 2
5*cos(d*x + c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x +
c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(5/2*d*x + 5/2*c) + 70*(2*
(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x
 + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c)
+ 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x
 + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c)
+ 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*c
os(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x
+ 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x +
5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*
d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c)
)*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*
sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*co
s(7/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 70*(2*(5*cos(4
*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) +
1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos
(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^
2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*co
s(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x
 + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) +
 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 +
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 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*
c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*
d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*
x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(3/5*ar
ctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 15*(2*(5*cos(4*d*x + 4
*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5
*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x +
 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(
10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x
+ 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^
2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2
*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*s
in(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*
sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*
c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)
*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 256*((10*cos(2*d*x + 2*c) + 5*
cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) + cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2
*c) + 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) + 10*cos(3*d*x + 3*c)*sin(5/2
*d*x + 5/2*c))*sin(5*d*x + 5*c) + 6400*((2*sin(2*d*x + 2*c) + sin(d*x + c))
*cos(5/2*d*x + 5/2*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c))*sin(4*d*x
+ 4*c) - 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) +
 cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x +
 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5
*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/
2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/
2*d*x + 5/2*c))*sin(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))
) - 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(
5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c)
 + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(
d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c)
- 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x
 + 5/2*c))*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 1
280*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d
*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10
*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x +
 c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*c
os(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/
2*c))*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sin(8/5
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 56*((10*cos(4*d*x +
 4*c) + 20*cos(3*d*x + 3*c) + 20*cos(2*d*x + 2*c) + 10*cos(d*x + c) + 3)*co
s(5*d*x + 5*c)^2 + cos(5*d*x + 5*c)^3 + (cos(5*d*x + 5*c) + 1)*sin(5*d*x +
5*c)^2 + (10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) +
2)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*c
os(d*x + c) + 2)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x
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+ c) + 2)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 5
0*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c)
 + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*
x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x
+ 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 20*cos(d*x + c) + 3)*cos(5*d*x +
5*c) + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*
cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(
d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c
) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(
2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d
*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c
) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d
*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c
) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100
*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*
x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x
+ 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(
4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x +
c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 10
0*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*
cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*(2*(5*cos
(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c)
+ 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*c
os(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c
)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*
cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d
*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c)
 + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2
 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x +
4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(
3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*
d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(4/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*(2*(5*cos(4*d*x +
4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(
5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x
+ 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*
(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x
 + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)
^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(
2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*
sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25
*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3
*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c
)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(2/5*arctan2(s
in(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*((sin(4*d*x + 4*c) + 2*sin
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(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5*d*x + 5*c) + sin(4
*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5
*d*x + 5*c) + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*s
in(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x
 + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 +
 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) +
1)*sin(7/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 8*(128*co
s(5*d*x + 5*c)^2*cos(5/2*d*x + 5/2*c) + 3200*cos(4*d*x + 4*c)^2*cos(5/2*d*x
 + 5/2*c) + 2560*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c
)*cos(5/2*d*x + 5/2*c) + 12800*cos(3*d*x + 3*c)^2*cos(5/2*d*x + 5/2*c) - 12
8*cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c)^2 + 3200*cos(5/2*d*x + 5/2*c)*sin(4
*d*x + 4*c)^2 + 12800*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2
*c)*sin(3*d*x + 3*c) + 12800*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c)^2 + 256*
((10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5/2*d*x + 5/2*c) + 5*cos(4*
d*x + 4*c)*cos(5/2*d*x + 5/2*c) + 10*cos(3*d*x + 3*c)*cos(5/2*d*x + 5/2*c))
*cos(5*d*x + 5*c) + 1280*((10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5/
2*d*x + 5/2*c) + 10*cos(3*d*x + 3*c)*cos(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c)
 + 128*(20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 +
 25*cos(d*x + c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x
+ c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(5/2*d*x + 5/2*c) - 70*(
2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d
*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c
) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d
*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c
) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100
*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*
x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x
+ 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(
4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x +
c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 10
0*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*
cos(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 15*(2*(5*cos
(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c)
+ 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*c
os(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c
)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*
cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d
*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c)
 + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2
 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x +
4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(
3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*
d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(1/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 256*((10*cos(2*d*x +
 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) + cos(5*d*x + 5*c)*sin(5/2
```

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*d*x + 5/2*c) + 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) + 10*cos(3*d*x + 3*
c)*sin(5/2*d*x + 5/2*c))*sin(5*d*x + 5*c) + 6400*((2*sin(2*d*x + 2*c) + sin
(d*x + c))*cos(5/2*d*x + 5/2*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c))*
sin(4*d*x + 4*c) - 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x
+ 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*s
in(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x
+ 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/
2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3
*c)*sin(5/2*d*x + 5/2*c))*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
 + 5/2*c))) - 1280*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2
*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*
d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c
) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x
 + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*s
in(5/2*d*x + 5/2*c))*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/
2*c))))*sin(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 8*(1
28*cos(5*d*x + 5*c)^2*cos(5/2*d*x + 5/2*c) + 3200*cos(4*d*x + 4*c)^2*cos(5/
2*d*x + 5/2*c) + 2560*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x
+ 3*c)*cos(5/2*d*x + 5/2*c) + 12800*cos(3*d*x + 3*c)^2*cos(5/2*d*x + 5/2*c)
 - 128*cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c)^2 + 3200*cos(5/2*d*x + 5/2*c)*
sin(4*d*x + 4*c)^2 + 12800*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x
+ 5/2*c)*sin(3*d*x + 3*c) + 12800*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c)^2 +
 256*((10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5/2*d*x + 5/2*c) + 5*c
os(4*d*x + 4*c)*cos(5/2*d*x + 5/2*c) + 10*cos(3*d*x + 3*c)*cos(5/2*d*x + 5/
2*c))*cos(5*d*x + 5*c) + 1280*((10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*c
os(5/2*d*x + 5/2*c) + 10*cos(3*d*x + 3*c)*cos(5/2*d*x + 5/2*c))*cos(4*d*x +
 4*c) + 128*(20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c
)^2 + 25*cos(d*x + c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin
(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(5/2*d*x + 5/2*c) -
 70*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*
cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x
+ 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*co
s(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x
+ 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c)
+ 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin
(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5
*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))
*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d
*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2
 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c)
+ 1)*cos(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 15*(2*(
5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x
+ c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) +
 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x
+ 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) +
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 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*co
s(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x +
 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5
*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d
*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))
*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*s
in(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos
(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 256*((10*cos(2*
d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) + cos(5*d*x + 5*c)*si
n(5/2*d*x + 5/2*c) + 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) + 10*cos(3*d*x
 + 3*c)*sin(5/2*d*x + 5/2*c))*sin(5*d*x + 5*c) + 6400*((2*sin(2*d*x + 2*c)
+ sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3
*c))*sin(4*d*x + 4*c) - 1280*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2
*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2
*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2
*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*s
in(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*
x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/
2*d*x + 5/2*c))))*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
))) - 56*((10*cos(4*d*x + 4*c) + 20*cos(3*d*x + 3*c) + 20*cos(2*d*x + 2*c)
+ 10*cos(d*x + c) + 3)*cos(5*d*x + 5*c)^2 + cos(5*d*x + 5*c)^3 + (cos(5*d*x
 + 5*c) + 1)*sin(5*d*x + 5*c)^2 + (10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x +
 2*c) + 5*cos(d*x + c) + 2)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(
10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 2)*cos(3*d*x + 3*c) + 100*cos(3*d*x
+ 3*c)^2 + 20*(5*cos(d*x + c) + 2)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^
2 + 25*cos(d*x + c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d
*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c)
 + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x
+ 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 20*cos(d
*x + c) + 3)*cos(5*d*x + 5*c) + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*
c) + 5*cos(d*x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*
cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3
*c)^2 + 20*(5*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 +
 25*cos(d*x + c)^2 + 5*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d*x + 3*c) + 10*co
s(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + cos(5*d*x + 5*c)^2
+ 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(4
*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d*x +
 c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c) + 1
)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*(sin(4
*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5
*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2
*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2
*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100
*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2
 + 10*cos(d*x + c) + 1)*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
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 5/2*c))) + 10*((sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c)
 + sin(d*x + c))*cos(5*d*x + 5*c) + sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) +
 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sin(3*d*x + 3*
c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4
*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(
3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c)
 + 25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*sin(3/5*arctan2(sin(5/2*d*x + 5
/2*c), cos(5/2*d*x + 5/2*c))) - 4*(128*cos(5*d*x + 5*c)^2*cos(5/2*d*x + 5/2
*c) + 3200*cos(4*d*x + 4*c)^2*cos(5/2*d*x + 5/2*c) + 2560*(10*cos(2*d*x + 2
*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c)*cos(5/2*d*x + 5/2*c) + 12800*cos
(3*d*x + 3*c)^2*cos(5/2*d*x + 5/2*c) - 128*cos(5/2*d*x + 5/2*c)*sin(5*d*x +
 5*c)^2 + 3200*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c)^2 + 12800*(2*sin(2*d*x
 + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) + 12800*cos(5
/2*d*x + 5/2*c)*sin(3*d*x + 3*c)^2 + 256*((10*cos(2*d*x + 2*c) + 5*cos(d*x
+ c) + 1)*cos(5/2*d*x + 5/2*c) + 5*cos(4*d*x + 4*c)*cos(5/2*d*x + 5/2*c) +
10*cos(3*d*x + 3*c)*cos(5/2*d*x + 5/2*c))*cos(5*d*x + 5*c) + 1280*((10*cos(
2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5/2*d*x + 5/2*c) + 10*cos(3*d*x + 3*
c)*cos(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c) + 128*(20*(5*cos(d*x + c) + 1)*co
s(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 100*sin(2*d*x
 + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 10*cos(
d*x + c) + 1)*cos(5/2*d*x + 5/2*c) - 15*(2*(5*cos(4*d*x + 4*c) + 10*cos(3*d
*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*cos(5*d*x + 5*c) + co
s(5*d*x + 5*c)^2 + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*
x + c) + 1)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2
*c) + 5*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5
*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x +
 c)^2 + 10*(sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + si
n(d*x + c))*sin(5*d*x + 5*c) + sin(5*d*x + 5*c)^2 + 50*(2*sin(3*d*x + 3*c)
+ 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)
^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d
*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) +
25*sin(d*x + c)^2 + 10*cos(d*x + c) + 1)*cos(1/5*arctan2(sin(5/2*d*x + 5/2*
c), cos(5/2*d*x + 5/2*c))) + 256*((10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1
)*sin(5/2*d*x + 5/2*c) + cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) + 5*cos(4*d*
x + 4*c)*sin(5/2*d*x + 5/2*c) + 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*s
in(5*d*x + 5*c) + 6400*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5
/2*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c))*sin(4*d*x + 4*c))*sin(2/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 12*((10*cos(4*d*x +
4*c) + 20*cos(3*d*x + 3*c) + 20*cos(2*d*x + 2*c) + 10*cos(d*x + c) + 3)*cos
(5*d*x + 5*c)^2 + cos(5*d*x + 5*c)^3 + (cos(5*d*x + 5*c) + 1)*sin(5*d*x + 5
*c)^2 + (10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 2
)*cos(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*co
s(d*x + c) + 2)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x +
 c) + 2)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 50
*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(4*d*x + 4*c)
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+ 25*sin(4*d*x + 4*c)^2 + 100*(2*sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x
 + 3*c) + 100*sin(3*d*x + 3*c)^2 + 100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x +
 2*c)*sin(d*x + c) + 25*sin(d*x + c)^2 + 20*cos(d*x + c) + 3)*cos(5*d*x + 5
*c) + 10*(10*cos(3*d*x + 3*c) + 10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*c
os(4*d*x + 4*c) + 25*cos(4*d*x + 4*c)^2 + 20*(10*cos(2*d*x + 2*c) + 5*cos(d
*x + c) + 1)*cos(3*d*x + 3*c) + 100*cos(3*d*x + 3*c)^2 + 20*(5*cos(d*x + c)
 + 1)*cos(2*d*x + 2*c) + 100*cos(2*d*x + 2*c)^2 + 25*cos(d*x + c)^2 + 10*((
sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*c) + sin(d*x + c))*
cos(5*d*x + 5*c) + sin(4*d*x + 4*c) + 2*sin(3*d*x + 3*c) + 2*sin(2*d*x + 2*
c) + sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sin(3*d*x + 3*c) + 2*sin(2*d*x
+ 2*c) + sin(d*x + c))*sin(4*d*x + 4*c) + 25*sin(4*d*x + 4*c)^2 + 100*(2*si
n(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + 100*sin(3*d*x + 3*c)^2 +
100*sin(2*d*x + 2*c)^2 + 100*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sin(d*x + c
)^2 + 10*cos(d*x + c) + 1)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*
x + 5/2*c))))/((sqrt(2)*a^2*cos(5*d*x + 5*c)^4 + sqrt(2)*a^2*sin(5*d*x + 5*
c)^4 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)
^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + 2
5*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100
*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x
+ c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 2*(5*sqrt(2)*a^2*cos(4*d*x + 4*c) +
10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(
2)*a^2*cos(d*x + c) + 2*sqrt(2)*a^2)*cos(5*d*x + 5*c)^3 + 10*(sqrt(2)*a^2*s
in(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x
+ 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(5*d*x + 5*c)^3 + 10*sqrt(2)*a^2*cos(
d*x + c) + (25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x +
 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^
2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2
+ 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin
(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 30*sqrt(2)*a^2*cos(d*x + c) + 6
*sqrt(2)*a^2 + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d
*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + 3*sqrt(2)*a^2)*cos(4*d*x + 4*c) +
20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + 3*sqrt(2
)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + 3*sqrt(2)*a^2)*c
os(2*d*x + 2*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*
d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^
2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*cos(5*d*x
+ 5*c)^2 + 25*(sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*cos(4*d*x +
4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2
*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(5*d*x + 5*c)^2 + 25
*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*
sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x +
 c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)
*a^2 + 2*(5*sqrt(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(2)*a^2*cos(3*d*x + 3*c)
+ 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^
2)*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*
```

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cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*
c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqr
t(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*
cos(2*d*x + 2*c) + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d
*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(
5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x
 + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*s
in(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*cos(8/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 100*(sqrt(2)*a^2*cos(5*
d*x + 5*c)^2 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*
x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x +
c)^2 + sqrt(2)*a^2*sin(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 +
 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 +
100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)
^2 + 10*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*cos(4*d*x
 + 4*c) + 10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c)
 + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(5*d*x + 5*c) + 10*(10*sqrt
(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*
cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x
 + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(
5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 10*(sqrt(2)*a^
2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d
*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*
sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c
))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin
(d*x + c))*sin(3*d*x + 3*c))*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))^2 + 100*(sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*cos
(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(
2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(5*d*x + 5*
c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)
^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*
sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c)
+ sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(2)*a^2*cos(3*d*
x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + s
qrt(2)*a^2)*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqr
t(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4
*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x +
 c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt
(2)*a^2)*cos(2*d*x + 2*c) + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^
2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x +
 c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*
sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt
(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*cos(
4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 25*(sqrt(2)*a^
2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*
```

```
cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*co
s(d*x + c)^2 + sqrt(2)*a^2*sin(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*sin(4*d*x +
4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2
*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(
d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*c
os(4*d*x + 4*c) + 10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*
x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(5*d*x + 5*c) + 10*
(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt
(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*c
os(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c
) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 10*(sq
rt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2
*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sqrt
(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin
(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)
*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c),
cos(5/2*d*x + 5/2*c)))^2 + (2*sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a
^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^
2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + 25*sqrt(2)*a^2*sin(4
*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*
d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a
^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c) + 2*sqrt(2)*a^2 + 2*(5*sqrt
(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2
*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + 2*sqrt(2)*a^2)*cos(5*d*x +
 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*
c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sq
rt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(
3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*
c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) +
sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x +
 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*sin(5*d*x + 5*c)^2 + 25
*(sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*
sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sq
rt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*
sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*s
in(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt
(2)*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*s
qrt(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*
a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(5*d*x
+ 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2
*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*s
qrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos
(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2
*c) + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2
*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(5*d*x + 5*c)
```

```
+ 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqr
t(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*
c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*sin(8/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 100*(sqrt(2)*a^2*cos(5*d*x + 5*c)^2
+ 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 +
100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2
)*a^2*sin(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*
a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a
^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 10*sqrt(
2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*cos(4*d*x + 4*c) + 10*
sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*
a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*cos(3
*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c)
+ sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*s
qrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2
*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 10*(sqrt(2)*a^2*sin(4*d*x +
 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + s
qrt(2)*a^2*sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3
*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x
+ 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*si
n(3*d*x + 3*c))*sin(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))
)^2 + 100*(sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)
^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^
2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(5*d*x + 5*c)^2 + 25*sqr
t(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt
(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c)
+ 25*sqrt(2)*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2
 + 2*(5*sqrt(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10
*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*c
os(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(
2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) +
 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)
*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(
2*d*x + 2*c) + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x +
 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(5*d*
x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2
*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2
*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*sin(4/5*arctan2(s
in(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 25*(sqrt(2)*a^2*cos(5*d*x +
 5*c)^2 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3
*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2
+ sqrt(2)*a^2*sin(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*
sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*s
qrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 +
10*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*cos(4*d*x + 4*
```

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c) + 10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*
sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a
^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d
*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*
c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqr
t(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 10*(sqrt(2)*a^2*sin
(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x +
2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3
*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*si
n(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x
+ c))*sin(3*d*x + 3*c))*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
 5/2*c)))^2 + sqrt(2)*a^2 + 2*(25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt
(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2
)*a^2*cos(d*x + c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*
sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*s
in(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 15*sqrt(2)*a
^2*cos(d*x + c) + 2*sqrt(2)*a^2 + 5*(20*sqrt(2)*a^2*cos(3*d*x + 3*c) + 20*s
qrt(2)*a^2*cos(2*d*x + 2*c) + 10*sqrt(2)*a^2*cos(d*x + c) + 3*sqrt(2)*a^2)*
cos(4*d*x + 4*c) + 10*(20*sqrt(2)*a^2*cos(2*d*x + 2*c) + 10*sqrt(2)*a^2*cos
(d*x + c) + 3*sqrt(2)*a^2)*cos(3*d*x + 3*c) + 10*(10*sqrt(2)*a^2*cos(d*x +
c) + 3*sqrt(2)*a^2)*cos(2*d*x + 2*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) +
 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c
) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d
*x + 3*c))*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt
(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*
d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x +
c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt(
2)*a^2)*cos(2*d*x + 2*c) + 10*(sqrt(2)*a^2*cos(5*d*x + 5*c)^3 + 25*sqrt(2)*
a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a
^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + 25*sqrt(2)*a^2*sin(
4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2
*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*
a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c) + (10*sqrt(2)*a^2*cos(4*d*
x + 4*c) + 20*sqrt(2)*a^2*cos(3*d*x + 3*c) + 20*sqrt(2)*a^2*cos(2*d*x + 2*c
) + 10*sqrt(2)*a^2*cos(d*x + c) + 3*sqrt(2)*a^2)*cos(5*d*x + 5*c)^2 + (sqrt
(2)*a^2*cos(5*d*x + 5*c) + sqrt(2)*a^2)*sin(5*d*x + 5*c)^2 + sqrt(2)*a^2 +
(25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 1
00*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + 25*sqrt
(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(
2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) +
 25*sqrt(2)*a^2*sin(d*x + c)^2 + 20*sqrt(2)*a^2*cos(d*x + c) + 3*sqrt(2)*a^
2 + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) +
 5*sqrt(2)*a^2*cos(d*x + c) + 2*sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt
(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + 2*sqrt(2)*a^2)*cos(
3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + 2*sqrt(2)*a^2)*cos(2*d*x +
```

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2*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c)
+ sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x
 + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*cos(5*d*x + 5*c) + 10
*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqr
t(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*
cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*
c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 10*(s
qrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqr
t(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(
2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*sin
(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(
2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)
*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sqrt
(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2
*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(5*d*x + 5
*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c)
 + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt
(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*
d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c)
 + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sq
rt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(5*d*x + 5*c) + 5
0*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2
)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c)
+ sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*cos(6/5*arctan2(sin(5/2*d*x +
 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*(sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*s
qrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sq
rt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*
sin(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*si
n(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin
(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2
*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(2
)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*co
s(d*x + c) + sqrt(2)*a^2)*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x +
 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt
(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)
*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d
*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c)
+ 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)
*a^2*sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) +
2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c)
 + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*
x + 3*c))*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*
(sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*s
qrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqr
t(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*s
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in(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*si
n(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(
2)*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sq
rt(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a
^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(5*d*x +
 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*
c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sq
rt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(
3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*
c) + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*
sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(5*d*x + 5*c) +
 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt
(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c
) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*cos(2/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sq
rt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*s
in(d*x + c) + (sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c
) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*cos(5*d*x +
5*c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2
*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqr
t(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*cos
(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*(sqrt(2)*a^2
*cos(5*d*x + 5*c)^3 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*c
os(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos
(d*x + c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x
 + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x
+ 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*
x + c) + (10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 20*sqrt(2)*a^2*cos(3*d*x + 3*c)
 + 20*sqrt(2)*a^2*cos(2*d*x + 2*c) + 10*sqrt(2)*a^2*cos(d*x + c) + 3*sqrt(2
)*a^2)*cos(5*d*x + 5*c)^2 + (sqrt(2)*a^2*cos(5*d*x + 5*c) + sqrt(2)*a^2)*si
n(5*d*x + 5*c)^2 + sqrt(2)*a^2 + (25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*s
qrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqr
t(2)*a^2*cos(d*x + c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a
^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^
2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 20*sqrt(2
)*a^2*cos(d*x + c) + 3*sqrt(2)*a^2 + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) +
10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + 2*sqrt(2)*a^
2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*c
os(d*x + c) + 2*sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x +
 c) + 2*sqrt(2)*a^2)*cos(2*d*x + 2*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c)
+ 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*
c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*
d*x + 3*c))*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqr
t(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4
*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x +

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 c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt
(2)*a^2)*cos(2*d*x + 2*c) + 10*(sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)
*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*
a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(5*
d*x + 5*c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*
x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x
 + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d
*x + c) + sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(2)*a^2*
cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x
+ c) + sqrt(2)*a^2)*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c)
+ 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^
2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*c
os(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c
) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sq
rt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*s
in(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt
(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100
*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*
c))*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*(sqrt(
2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)
*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a
^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*sin(4*d
*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*
x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2
*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sqrt(2)*
a^2*cos(4*d*x + 4*c) + 10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos
(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(5*d*x + 5*c)
+ 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5
*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*
a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x
+ 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 1
0*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2
)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2
*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^
2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sq
rt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*cos(2/5*arctan2(sin(5/2*d*x + 5/2
*c), cos(5/2*d*x + 5/2*c))) + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*
a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x
 + c) + (sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*
sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*cos(5*d*x + 5*c))*
sin(5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2
*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a
^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*cos(6/5*a
rctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*(sqrt(2)*a^2*cos(5
*d*x + 5*c)^3 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d
```

```
*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x +
 c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c
)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)
*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c)
 + (10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 20*sqrt(2)*a^2*cos(3*d*x + 3*c) + 20*
sqrt(2)*a^2*cos(2*d*x + 2*c) + 10*sqrt(2)*a^2*cos(d*x + c) + 3*sqrt(2)*a^2)
*cos(5*d*x + 5*c)^2 + (sqrt(2)*a^2*cos(5*d*x + 5*c) + sqrt(2)*a^2)*sin(5*d*
x + 5*c)^2 + sqrt(2)*a^2 + (25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)
*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a
^2*cos(d*x + c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin
(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(
2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 20*sqrt(2)*a^2*
cos(d*x + c) + 3*sqrt(2)*a^2 + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqr
t(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + 2*sqrt(2)*a^2)*cos
(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x
 + c) + 2*sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) +
2*sqrt(2)*a^2)*cos(2*d*x + 2*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sq
rt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 1
00*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x +
3*c))*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a
^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x +
 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) +
sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^
2)*cos(2*d*x + 2*c) + 5*(sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*co
s(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos
(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(5*d*x + 5
*c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c
)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)
*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c)
 + sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(2)*a^2*cos(3*d
*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) +
sqrt(2)*a^2)*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sq
rt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(
4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x
+ c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqr
t(2)*a^2)*cos(2*d*x + 2*c) + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a
^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x
+ c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2
*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqr
t(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*cos
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*(sqrt(2)*a^2
*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*
x + 2*c) + sqrt(2)*a^2*sin(d*x + c) + (sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqr
t(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*si
n(d*x + c))*cos(5*d*x + 5*c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3*d*
```

```
x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4
*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c
))*sin(3*d*x + 3*c))*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/
2*c))) + 10*(sqrt(2)*a^2*cos(5*d*x + 5*c)^3 + 25*sqrt(2)*a^2*cos(4*d*x + 4*
c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c
)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 1
00*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 10
0*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2
 + 10*sqrt(2)*a^2*cos(d*x + c) + (10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 20*sqrt
(2)*a^2*cos(3*d*x + 3*c) + 20*sqrt(2)*a^2*cos(2*d*x + 2*c) + 10*sqrt(2)*a^2
*cos(d*x + c) + 3*sqrt(2)*a^2)*cos(5*d*x + 5*c)^2 + (sqrt(2)*a^2*cos(5*d*x
+ 5*c) + sqrt(2)*a^2)*sin(5*d*x + 5*c)^2 + sqrt(2)*a^2 + (25*sqrt(2)*a^2*co
s(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos
(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + 25*sqrt(2)*a^2*sin(4*d*x
+ 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x +
 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*si
n(d*x + c)^2 + 20*sqrt(2)*a^2*cos(d*x + c) + 3*sqrt(2)*a^2 + 10*(10*sqrt(2)
*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos
(d*x + c) + 2*sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x
+ 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + 2*sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*
(5*sqrt(2)*a^2*cos(d*x + c) + 2*sqrt(2)*a^2)*cos(2*d*x + 2*c) + 50*(2*sqrt(
2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(
d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*
a^2*sin(d*x + c))*sin(3*d*x + 3*c))*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*c
os(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x +
 c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) +
 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)
*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 10*(sqrt(2)*a^2*sin(4*d
*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c)
 + sqrt(2)*a^2*sin(d*x + c) + (sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2
*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x +
c))*cos(5*d*x + 5*c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c)
 + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4
*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3
*d*x + 3*c))*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt
(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c) + (sqrt(2)*a^2*sin(4*d*
x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c)
+ sqrt(2)*a^2*sin(d*x + c))*cos(5*d*x + 5*c)^2 + 2*(sqrt(2)*a^2*sin(4*d*x +
 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + s
qrt(2)*a^2*sin(d*x + c))*cos(5*d*x + 5*c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2)
*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*
x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^
2*sin(d*x + c))*sin(3*d*x + 3*c) + 10*(sqrt(2)*a^2*sin(5*d*x + 5*c)^3 + 10*
(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*
```

```
a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(5*d*x + 5*c)^2 + (sqrt
(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2
)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*
a^2*cos(d*x + c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*si
n(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin
(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2
*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(2
)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*co
s(d*x + c) + sqrt(2)*a^2)*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x +
 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt
(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)
*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d
*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c
) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x +
4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(
3*d*x + 3*c))*sin(5*d*x + 5*c) + 10*(sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sq
rt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqr
t(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*s
in(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin
(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(
2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*
cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(2)
*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos
(d*x + c) + sqrt(2)*a^2)*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x +
3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(
2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*
a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*
x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) +
 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*
a^2*sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2
*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c)
+ 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x
 + 3*c))*sin(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*
(sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*s
qrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqr
t(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*s
in(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*si
n(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(
2)*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sq
rt(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a
^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(5*d*x +
 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*
c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sq
rt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(
3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*
```

c) + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*
sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(5*d*x + 5*c) +
 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt
(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c
) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*sin(4/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*(sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*
sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*s
qrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2
*sin(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*s
in(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*si
n(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^
2*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(
2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*c
os(d*x + c) + sqrt(2)*a^2)*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x
+ 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqr
t(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2
)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(
d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c)
 + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2
)*a^2*sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) +
 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c
) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d
*x + 3*c))*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*si
n(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*(sqrt(2)*a^
2*sin(5*d*x + 5*c)^3 + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin
(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*
sin(5*d*x + 5*c)^2 + (sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*cos(4
*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*
d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4
*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*
c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d
*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*co
s(4*d*x + 4*c) + 10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x
 + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(5*d*x + 5*c) + 10*(
10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(
2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*co
s(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c)
 + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 50*(2*s
qrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*
sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt
(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*sin(5*d*x + 5*c) + 10*(sqrt(2)*a^2*
cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*co
s(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(
d*x + c)^2 + sqrt(2)*a^2*sin(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*
c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c

```
)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*
x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*cos
(4*d*x + 4*c) + 10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x
+ 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(5*d*x + 5*c) + 10*(1
0*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2
)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos
(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c)
+ 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 10*(sqrt
(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*s
in(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2
)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d
*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a
^2*sin(d*x + c))*sin(3*d*x + 3*c))*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), co
s(5/2*d*x + 5/2*c))) + 5*(sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*c
os(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*co
s(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(5*d*x +
5*c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*
c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c
)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c
) + sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(2)*a^2*cos(3*
d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) +
 sqrt(2)*a^2)*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*s
qrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos
(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x
 + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sq
rt(2)*a^2)*cos(2*d*x + 2*c) + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*
a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x
 + c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^
2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sq
rt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sin(6/5*arctan2
(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*(sqrt(2)*a^2*sin(5*d*x +
 5*c)^3 + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c)
 + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(5*d*x + 5
*c)^2 + (sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2
 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2
+ 25*sqrt(2)*a^2*cos(d*x + c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*s
qrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sq
rt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 1
0*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*cos(4*d*x + 4*c
) + 10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*s
qrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^
2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*
x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c
) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt
```

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(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 50*(2*sqrt(2)*a^2*si
n(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))
*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d
*x + c))*sin(3*d*x + 3*c))*sin(5*d*x + 5*c) + 5*(sqrt(2)*a^2*cos(5*d*x + 5*
c)^2 + 25*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)
^2 + 100*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + s
qrt(2)*a^2*sin(5*d*x + 5*c)^2 + 25*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 100*sqr
t(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 100*sqrt
(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)*a^2*sin(d*x + c)^2 + 10*
sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sqrt(2)*a^2*cos(4*d*x + 4*c)
+ 10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqr
t(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(5*d*x + 5*c) + 10*(10*sqrt(2)*a^2*
cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x
+ c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt(2)*a^2*cos(2*d*x + 2*c)
+ 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 20*(5*sqrt(2
)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 10*(sqrt(2)*a^2*sin(4*
d*x + 4*c) + 2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c
) + sqrt(2)*a^2*sin(d*x + c))*sin(5*d*x + 5*c) + 50*(2*sqrt(2)*a^2*sin(3*d*
x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c))*sin(4
*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(d*x + c
))*sin(3*d*x + 3*c))*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/
2*c))))*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 10*(
sqrt(2)*a^2*sin(5*d*x + 5*c)^3 + 10*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(
2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(
d*x + c))*sin(5*d*x + 5*c)^2 + (sqrt(2)*a^2*cos(5*d*x + 5*c)^2 + 25*sqrt(2)
*a^2*cos(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 100*sqrt(2)*
a^2*cos(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(d*x + c)^2 + 25*sqrt(2)*a^2*sin
(4*d*x + 4*c)^2 + 100*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 100*sqrt(2)*a^2*sin(
2*d*x + 2*c)^2 + 100*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 25*sqrt(2)
*a^2*sin(d*x + c)^2 + 10*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(5*sqrt
(2)*a^2*cos(4*d*x + 4*c) + 10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2
*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(5*d*x + 5
*c) + 10*(10*sqrt(2)*a^2*cos(3*d*x + 3*c) + 10*sqrt(2)*a^2*cos(2*d*x + 2*c)
 + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 20*(10*sqrt
(2)*a^2*cos(2*d*x + 2*c) + 5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*
d*x + 3*c) + 20*(5*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c)
 + 50*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + sq
rt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 100*(2*sqrt(2)*a^2*sin(2*d*x + 2
*c) + sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*sin(5*d*x + 5*c))*sin(2/5
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqrt(a)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}\left(\frac{3\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{3\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{2\left(3\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\right.\right.}\right.}{64\sqrt{a}d}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(3*log(sin(1/2*d*x + 1/2*c) + 1)/(a^2*sgn(cos(1/2*d*x + 1/2*c)
)) - 3*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^2*sgn(cos(1/2*d*x + 1/2*c))) - 2*(
3*sin(1/2*d*x + 1/2*c)^3 - 5*sin(1/2*d*x + 1/2*c))/((sin(1/2*d*x + 1/2*c)^2
 - 1)^2*a^2*sgn(cos(1/2*d*x + 1/2*c)))))/(sqrt(a)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

```
[In] int(1/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(1/(a + a*cos(c + d*x))^(5/2), x)
```

3.8 $\int (a+a\cos(c+dx))^{4/3}\,dx$

Optimal result 131
Rubi [A] (verified) 131
Mathematica [A] (verified) 132
Maple [F] 132
Fricas [F] 133
Sympy [F] 133
Maxima [F] 133
Giac [F] 133
Mupad [F(-1)] 134

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int (a+a\cos(c+dx))^{4/3}\,dx=\frac{2\,2^{5/6}a\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{d(1+\cos(c+dx))^{5/6}}$$

```
[Out] 2*2^(5/6)*a*(a+a*cos(d*x+c))^(1/3)*hypergeom([-5/6, 1/2],[3/2],1/2-1/2*cos(
d*x+c))*sin(d*x+c)/d/(1+cos(d*x+c))^(5/6)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2731, 2730}

$$\int (a+a\cos(c+dx))^{4/3}\,dx=\frac{2\,2^{5/6}a\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{5/6}}$$

```
[In] Int[(a + a*Cos[c + d*x])^(4/3),x]
```

```
[Out] (2*2^(5/6)*a*(a + a*Cos[c + d*x])^(1/3)*Hypergeometric2F1[-5/6, 1/2, 3/2, (
1 - Cos[c + d*x])/2]*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(5/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
```

```
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt[3]{a+a\cos(c+dx)}\right)\int(1+\cos(c+dx))^{4/3}\,dx}{\sqrt[3]{1+\cos(c+dx)}}$$

$$= \frac{2\ 2^{5/6}a\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{d(1+\cos(c+dx))^{5/6}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int(a+a\cos(c+dx))^{4/3}\,dx =$$

$$-\frac{6(a(1+\cos(c+dx)))^{4/3}\cot\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}{11d}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(4/3),x]
[Out] (-6*(a*(1 + Cos[c + d*x]))^(4/3)*Cot[(c + d*x)/2]*Hypergeometric2F1[1/2, 11
/6, 17/6, Cos[(c + d*x)/2]^2]*Sqrt[Sin[(c + d*x)/2]^2])/(11*d)
```

Maple [F]

$$\int(a+\cos(dx+c)\,a)^{\frac{4}{3}}\,dx$$

```
[In] int((a+cos(d*x+c)*a)^(4/3),x)
[Out] int((a+cos(d*x+c)*a)^(4/3),x)
```

Fricas [F]

$$\int (a+a\cos(c+dx))^{4/3}\,dx=\int (a\cos(dx+c)+a)^{\frac{4}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((a*cos(d*x + c) + a)^(4/3), x)
```

Sympy [F]

$$\int (a+a\cos(c+dx))^{4/3}\,dx=\int (a\cos(c+dx)+a)^{\frac{4}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))**(4/3),x)
[Out] Integral((a*cos(c + d*x) + a)**(4/3), x)
```

Maxima [F]

$$\int (a+a\cos(c+dx))^{4/3}\,dx=\int (a\cos(dx+c)+a)^{\frac{4}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)^(4/3), x)
```

Giac [F]

$$\int (a+a\cos(c+dx))^{4/3}\,dx=\int (a\cos(dx+c)+a)^{\frac{4}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((a*cos(d*x + c) + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{4/3}\,dx = \int (a+a\,\cos{(c+d\,x)})^{4/3}\,dx$$

```
[In] int((a + a*cos(c + d*x))^(4/3),x)
[Out] int((a + a*cos(c + d*x))^(4/3), x)
```

3.9 $\int (a+a\cos(c+dx))^{2/3}\,dx$

Optimal result 135
Rubi [A] (verified) 135
Mathematica [A] (verified) 136
Maple [F] 136
Fricas [F] 137
Sympy [F] 137
Maxima [F] 137
Giac [F] 137
Mupad [F(-1)] 138

Optimal result

Integrand size = 14, antiderivative size = 66

$$\int (a+a\cos(c+dx))^{2/3}\,dx=\frac{2\sqrt[6]{2}(a+a\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{d(1+\cos(c+dx))^{7/6}}$$

```
[Out] 2*2^(1/6)*(a+a*cos(d*x+c))^(2/3)*hypergeom([-1/6, 1/2],[3/2],1/2-1/2*cos(d*
x+c))*sin(d*x+c)/d/(1+cos(d*x+c))^(7/6)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2731, 2730}

$$\int (a+a\cos(c+dx))^{2/3}\,dx=\frac{2\sqrt[6]{2}\sin(c+dx)(a\cos(c+dx)+a)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{7/6}}$$

```
[In] Int[(a + a*Cos[c + d*x])^(2/3),x]
```

```
[Out] (2*2^(1/6)*(a + a*Cos[c + d*x])^(2/3)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1
- Cos[c + d*x])/2]*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(7/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
```

```
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(a+a\cos(c+dx))^{2/3}\int(1+\cos(c+dx))^{2/3}\,dx}{(1+\cos(c+dx))^{2/3}}\\ &= \frac{2\sqrt[6]{2}(a+a\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{d(1+\cos(c+dx))^{7/6}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int(a+a\cos(c+dx))^{2/3}\,dx = -\frac{6(a(1+\cos(c+dx)))^{2/3}\cot\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}{7d}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(2/3),x]
[Out] (-6*(a*(1 + Cos[c + d*x]))^(2/3)*Cot[(c + d*x)/2]*Hypergeometric2F1[1/2, 7/
6, 13/6, Cos[(c + d*x)/2]^2]*Sqrt[Sin[(c + d*x)/2]^2])/(7*d)
```

Maple [F]

$$\int(a+\cos(dx+c)\,a)^{\frac{2}{3}}\,dx$$

```
[In] int((a+cos(d*x+c)*a)^(2/3),x)
[Out] int((a+cos(d*x+c)*a)^(2/3),x)
```

Fricas [F]

$$\int (a+a\cos(c+dx))^{2/3}\,dx = \int (a\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((a*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F]

$$\int (a+a\cos(c+dx))^{2/3}\,dx = \int (a\cos(c+dx)+a)^{\frac{2}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))**(2/3),x)
[Out] Integral((a*cos(c + d*x) + a)**(2/3), x)
```

Maxima [F]

$$\int (a+a\cos(c+dx))^{2/3}\,dx = \int (a\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)^(2/3), x)
```

Giac [F]

$$\int (a+a\cos(c+dx))^{2/3}\,dx = \int (a\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((a*cos(d*x + c) + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{2/3}\,dx = \int (a + a\ \cos(c + d\,x))^{2/3}\,dx$$

```
[In] int((a + a*cos(c + d*x))^(2/3),x)
[Out] int((a + a*cos(c + d*x))^(2/3), x)
```

3.10 $\int \sqrt[3]{a+a\cos(c+dx)}\,dx$

Optimal result 139
Rubi [A] (verified) 139
Mathematica [A] (verified) 140
Maple [F] 140
Fricas [F] 141
Sympy [F] 141
Maxima [F] 141
Giac [F] 141
Mupad [F(-1)] 142

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int \sqrt[3]{a+a\cos(c+dx)}\,dx = \frac{2^{5/6}\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{d(1+\cos(c+dx))^{5/6}}$$

```
[Out] 2^(5/6)*(a+a*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[3/2],1/2-1/2*cos(d*x+c
))*sin(d*x+c)/d/(1+cos(d*x+c))^(5/6)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2731, 2730}

$$\int \sqrt[3]{a+a\cos(c+dx)}\,dx = \frac{2^{5/6}\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{5/6}}$$

```
[In] Int[(a + a*Cos[c + d*x])^(1/3),x]
```

```
[Out] (2^(5/6)*(a + a*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - C
os[c + d*x])/2]*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(5/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
```

```
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{a+a\cos(c+dx)}\int\sqrt[3]{1+\cos(c+dx)}\,dx}{\sqrt[3]{1+\cos(c+dx)}}$$
$$= \frac{2^{5/6}\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{d(1+\cos(c+dx))^{5/6}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int\sqrt[3]{a+a\cos(c+dx)}\,dx =$$
$$-\frac{6\sqrt[3]{a(1+\cos(c+dx))}\cot\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}{5d}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(1/3),x]
[Out] (-6*(a*(1 + Cos[c + d*x]))^(1/3)*Cot[(c + d*x)/2]*Hypergeometric2F1[1/2, 5/
6, 11/6, Cos[(c + d*x)/2]^2]*Sqrt[Sin[(c + d*x)/2]^2])/(5*d)
```

Maple [F]

$$\int(a+\cos(dx+c)\,a)^{\frac{1}{3}}\,dx$$

```
[In] int((a+cos(d*x+c)*a)^(1/3),x)
[Out] int((a+cos(d*x+c)*a)^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\,dx = \int (a\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((a*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\,dx = \int \sqrt[3]{a\cos(c+dx)+a}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))**(1/3),x)
[Out] Integral((a*cos(c + d*x) + a)**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\,dx = \int (a\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\,dx = \int (a\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((a*cos(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+a\cos(c+dx)}\,dx = \int (a+a\,\cos(c+d\,x))^{1/3}\,dx$$

```
[In] int((a + a*cos(c + d*x))^(1/3),x)
[Out] int((a + a*cos(c + d*x))^(1/3), x)
```

3.11 $\int \frac{1}{\sqrt[3]{a+a\cos(c+dx)}}\,dx$

Optimal result . . . 143
Rubi [A] (verified) . . . 143
Mathematica [A] (verified) . . . 144
Maple [F] . . . 144
Fricas [F] . . . 145
Sympy [F] . . . 145
Maxima [F] . . . 145
Giac [F] . . . 145
Mupad [F(-1)] . . . 146

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int \frac{1}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \frac{\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{d\sqrt[6]{1+\cos(c+dx)}\sqrt[3]{a+a\cos(c+dx)}}$$

```
[Out] 2^(1/6)*hypergeom([1/2, 5/6],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)/d/(1+cos(
d*x+c))^(1/6)/(a+a*cos(d*x+c))^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2731, 2730}

$$\int \frac{1}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \frac{\sqrt[6]{2}\sin(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{d\sqrt[6]{\cos(c+dx)+1}\sqrt[3]{a\cos(c+dx)+a}}$$

```
[In] Int[(a + a*Cos[c + d*x])^(-1/3),x]
```

```
[Out] (2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x
])/(d*(1 + Cos[c + d*x])^(1/6)*(a + a*Cos[c + d*x])^(1/3))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt[3]{1+\cos(c+dx)}\int\frac{1}{\sqrt[3]{1+\cos(c+dx)}}\,dx}{\sqrt[3]{a+a\cos(c+dx)}}\\ &= \frac{\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{d\sqrt[6]{1+\cos(c+dx)}\sqrt[3]{a+a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int\frac{1}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = -\frac{6\cot\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}{d\sqrt[3]{a(1+\cos(c+dx))}}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(-1/3),x]
[Out] (-6*Cot[(c + d*x)/2]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[(c + d*x)/2]^2]*S
qrt[Sin[(c + d*x)/2]^2])/(d*(a*(1 + Cos[c + d*x]))^(1/3))
```

Maple [F]

$$\int\frac{1}{\left(a+\cos\left(dx+c\right)a\right)^{\frac{1}{3}}}dx$$

```
[In] int(1/(a+cos(d*x+c)*a)^(1/3),x)
[Out] int(1/(a+cos(d*x+c)*a)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{a+a\cos(c+dx)}}\,dx=\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((a*cos(d*x + c) + a)^(-1/3), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+a\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt[3]{a\cos\left(c+dx\right)+a}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))**(1/3),x)
[Out] Integral((a*cos(c + d*x) + a)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+a\cos(c+dx)}}\,dx=\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+a\cos(c+dx)}}\,dx=\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((a*cos(d*x + c) + a)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+a\cos(c+dx)}}\,dx=\int \frac{1}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

```
[In] int(1/(a + a*cos(c + d*x))^(1/3),x)
[Out] int(1/(a + a*cos(c + d*x))^(1/3), x)
```

3.12 $\int \frac{1}{(a+a\cos(c+dx))^{2/3}}\,dx$

Optimal result 147
Rubi [A] (verified) 147
Mathematica [A] (verified) 148
Maple [F] 148
Fricas [F] 149
Sympy [F] 149
Maxima [F] 149
Giac [F] 149
Mupad [F(-1)] 150

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int \frac{1}{(a+a\cos(c+dx))^{2/3}}\,dx = \frac{\sqrt[6]{1+\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+d}{\sqrt[6]{2}d(a+a\cos(c+dx))^{2/3}}$$

```
[Out] 1/2*(1+cos(d*x+c))^(1/6)*hypergeom([1/2, 7/6],[3/2],1/2-1/2*cos(d*x+c))*sin
(d*x+c)*2^(5/6)/d/(a+a*cos(d*x+c))^(2/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2731, 2730}

$$\int \frac{1}{(a+a\cos(c+dx))^{2/3}}\,dx = \frac{\sin(c+dx)\sqrt[6]{\cos(c+dx)+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)\right.}{\sqrt[6]{2}d(a\cos(c+dx)+a)^{2/3}}$$

```
[In] Int[(a + a*Cos[c + d*x])^(-2/3),x]
```

```
[Out] ((1 + Cos[c + d*x])^(1/6)*Hypergeometric2F1[1/2, 7/6, 3/2, (1 - Cos[c + d*x
])/2]*Sin[c + d*x])/(2^(1/6)*d*(a + a*Cos[c + d*x])^(2/3))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{(1+\cos(c+dx))^{2/3}\int \frac{1}{(1+\cos(c+dx))^{2/3}}\,dx}{(a+a\cos(c+dx))^{2/3}}$$

$$= \frac{\sqrt[6]{1+\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{\sqrt[6]{2}d(a+a\cos(c+dx))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+a\cos(c+dx))^{2/3}}\,dx = \frac{6\cot\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}{d(a(1+\cos(c+dx)))^{2/3}}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(-2/3),x]
[Out] (6*Cot[(c + d*x)/2]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[(c + d*x)/2]^2]*S
qrt[Sin[(c + d*x)/2]^2])/(d*(a*(1 + Cos[c + d*x]))^(2/3))
```

Maple [F]

$$\int \frac{1}{(a+\cos(dx+c)\,a)^{\frac{2}{3}}}dx$$

```
[In] int(1/(a+cos(d*x+c)*a)^(2/3),x)
[Out] int(1/(a+cos(d*x+c)*a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((a*cos(d*x + c) + a)^(-2/3), x)
```

Sympy [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{1}{(a\cos(c+dx)+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))**(2/3),x)
[Out] Integral((a*cos(c + d*x) + a)**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((a*cos(d*x + c) + a)^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{1}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

```
[In] int(1/(a + a*cos(c + d*x))^(2/3),x)
[Out] int(1/(a + a*cos(c + d*x))^(2/3), x)
```

3.13 $\int \frac{1}{(a+a\cos(c+dx))^{4/3}}\,dx$

Optimal result . . . 151
Rubi [A] (verified) . . . 151
Mathematica [A] (verified) . . . 152
Maple [F] . . . 152
Fricas [F] . . . 153
Sympy [F] . . . 153
Maxima [F] . . . 153
Giac [F] . . . 153
Mupad [F(-1)] . . . 154

Optimal result

Integrand size = 14, antiderivative size = 68

$$\int \frac{1}{(a+a\cos(c+dx))^{4/3}}\,dx=\frac{\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{2^{5/6}ad\sqrt[6]{1+\cos(c+dx)}\sqrt[3]{a+a\cos(c+dx)}}$$

```
[Out] 1/2*hypergeom([1/2, 11/6],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(1/6)/a/d/
(1+cos(d*x+c))^(1/6)/(a+a*cos(d*x+c))^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2731, 2730}

$$\int \frac{1}{(a+a\cos(c+dx))^{4/3}}\,dx=\frac{\sin(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\cos(c+dx)+1}\sqrt[3]{a\cos(c+dx)+a}}$$

```
[In] Int[(a + a*Cos[c + d*x])^(-4/3),x]
```

```
[Out] (Hypergeometric2F1[1/2, 11/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(2^(
5/6)*a*d*(1 + Cos[c + d*x])^(1/6)*(a + a*Cos[c + d*x])^(1/3))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt[3]{1+\cos(c+dx)}\int \frac{1}{(1+\cos(c+dx))^{4/3}}\,dx}{a\sqrt[3]{a+a\cos(c+dx)}}\\ &= \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{2^{5/6}ad\sqrt[6]{1+\cos(c+dx)}\sqrt[3]{a+a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+a\cos(c+dx))^{4/3}}\,dx = \frac{6\cot\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\sin^2\left(\frac{1}{2}(c+\right.}}{5d(a(1+\cos(c+dx)))^{4/3}}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(-4/3),x]
[Out] (6*Cot[(c + d*x)/2]*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[(c + d*x)/2]^2]*S
qrt[Sin[(c + d*x)/2]^2])/(5*d*(a*(1 + Cos[c + d*x]))^(4/3))
```

Maple [F]

$$\int \frac{1}{(a+\cos(dx+c)\,a)^{\frac{4}{3}}}dx$$

```
[In] int(1/(a+cos(d*x+c)*a)^(4/3),x)
[Out] int(1/(a+cos(d*x+c)*a)^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((a*cos(d*x + c) + a)^(2/3)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c
) + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{(a\cos(c+dx)+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))**(4/3),x)
[Out] Integral((a*cos(c + d*x) + a)**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((a*cos(d*x + c) + a)^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{(a+a\,\cos\left(c+d\,x\right))^{4/3}}\,dx$$

```
[In] int(1/(a + a*cos(c + d*x))^(4/3),x)
[Out] int(1/(a + a*cos(c + d*x))^(4/3), x)
```

3.14 $\int (a+a\cos(c+dx))^n\,dx$

Optimal result 155
Rubi [A] (verified) 155
Mathematica [A] (verified) 156
Maple [F] 156
Fricas [F] 157
Sympy [F] 157
Maxima [F] 157
Giac [F] 157
Mupad [F(-1)] 158

Optimal result

Integrand size = 12, antiderivative size = 73

$$\int (a+a\cos(c+dx))^n\,dx = \frac{2^{\frac{1}{2}+n}(1+\cos(c+dx))^{-\frac{1}{2}-n}(a+a\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(}{d}$$

```
[Out] 2^(1/2+n)*(1+cos(d*x+c))^(-1/2-n)*(a+a*cos(d*x+c))^n*hypergeom([1/2, 1/2-n]
,[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2731, 2730}

$$\int (a+a\cos(c+dx))^n\,dx = \frac{2^{n+\frac{1}{2}}\sin(c+dx)(\cos(c+dx)+1)^{-n-\frac{1}{2}}(a\cos(c+dx)+a)^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-\cos(c\right.}{d}$$

```
[In] Int[(a + a*Cos[c + d*x])^n,x]
```

```
[Out] (2^(1/2 + n)*(1 + Cos[c + d*x])^(-1/2 - n)*(a + a*Cos[c + d*x])^n*Hypergeom
etric2F1[1/2, 1/2 - n, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/d
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
```

```
b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \left((1+\cos(c+dx))^{-n}(a+a\cos(c+dx))^n\right)\int(1+\cos(c+dx))^n\,dx$$

$$= \frac{2^{\frac{1}{2}+n}(1+\cos(c+dx))^{-\frac{1}{2}-n}(a+a\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)\right.}{d}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int(a+a\cos(c+dx))^n\,dx =$$

$$-\frac{2(a(1+\cos(c+dx)))^n\cot\left(\frac{1}{2}(c+dx)\right)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+n,\frac{3}{2}+n,\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\sin^2\left(\frac{1}{2}(\right.}}{d+2dn}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^n,x]
[Out] (-2*(a*(1 + Cos[c + d*x]))^n*Cot[(c + d*x)/2]*Hypergeometric2F1[1/2, 1/2 +
n, 3/2 + n, Cos[(c + d*x)/2]^2]*Sqrt[Sin[(c + d*x)/2]^2])/(d + 2*d*n)
```

Maple [F]

$$\int(a+\cos(dx+c)\,a)^n\,dx$$

```
[In] int((a+cos(d*x+c)*a)^n,x)
[Out] int((a+cos(d*x+c)*a)^n,x)
```

Fricas [F]

$$\int (a + a\cos(c + dx))^n\,dx = \int (a\cos(dx + c) + a)^n\,dx$$

[In] `integrate((a+a*cos(d*x+c))^n,x, algorithm="fricas")`

[Out] `integral((a*cos(d*x + c) + a)^n, x)`

Sympy [F]

$$\int (a + a\cos(c + dx))^n\,dx = \int (a\cos(c + dx) + a)^n\,dx$$

[In] `integrate((a+a*cos(d*x+c))**n,x)`

[Out] `Integral((a*cos(c + d*x) + a)**n, x)`

Maxima [F]

$$\int (a + a\cos(c + dx))^n\,dx = \int (a\cos(dx + c) + a)^n\,dx$$

[In] `integrate((a+a*cos(d*x+c))^n,x, algorithm="maxima")`

[Out] `integrate((a*cos(d*x + c) + a)^n, x)`

Giac [F]

$$\int (a + a\cos(c + dx))^n\,dx = \int (a\cos(dx + c) + a)^n\,dx$$

[In] `integrate((a+a*cos(d*x+c))^n,x, algorithm="giac")`

[Out] `integrate((a*cos(d*x + c) + a)^n, x)`

Mupad [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^n \, dx = \int (a + a\,\cos(c + d\,x))^n \, dx$$

```
[In] int((a + a*cos(c + d*x))^n,x)
[Out] int((a + a*cos(c + d*x))^n, x)
```

3.15 $\int (a-a\cos(c+dx))^n\,dx$

Optimal result 159
Rubi [A] (verified) 159
Mathematica [A] (verified) 160
Maple [F] 160
Fricas [F] 161
Sympy [F] 161
Maxima [F] 161
Giac [F] 161
Mupad [F(-1)] 162

Optimal result

Integrand size = 13, antiderivative size = 75

$$\int (a-a\cos(c+dx))^n\,dx = -\frac{2^{\frac{1}{2}+n}(1-\cos(c+dx))^{-\frac{1}{2}-n}(a-a\cos(c+dx))^n \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1+\cos(c+dx))\right)\text{s}}{d}$$

```
[Out] -2^(1/2+n)*(1-cos(d*x+c))^(-1/2-n)*(a-a*cos(d*x+c))^n*hypergeom([1/2, 1/2-n
],[3/2],1/2+1/2*cos(d*x+c))*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2731, 2730}

$$\int (a-a\cos(c+dx))^n\,dx = -\frac{2^{n+\frac{1}{2}}\sin(c+dx)(1-\cos(c+dx))^{-n-\frac{1}{2}}(a-a\cos(c+dx))^n \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(\cos(c+\right.}{d}$$

```
[In] Int[(a - a*Cos[c + d*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*(1 - Cos[c + d*x])^(-1/2 - n)*(a - a*Cos[c + d*x])^n*Hyperge
ometric2F1[1/2, 1/2 - n, 3/2, (1 + Cos[c + d*x])/2]*Sin[c + d*x])/d)
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
```

```
b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \left((1-\cos(c+dx))^{-n}(a-a\cos(c+dx))^n\right)\int(1-\cos(c+dx))^n\,dx$$
$$= -\frac{2^{\frac{1}{2}+n}(1-\cos(c+dx))^{-\frac{1}{2}-n}(a-a\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1+\cos(c+dx\right.}{d}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int(a-a\cos(c+dx))^n\,dx$$
$$= \frac{\sqrt{2}\sqrt{1+\cos(c+dx)}(a-a\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+n,\frac{3}{2}+n,\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c\right.}{d+2dn}$$

```
[In] Integrate[(a - a*Cos[c + d*x])^n,x]
[Out] (Sqrt[2]*Sqrt[1 + Cos[c + d*x]]*(a - a*Cos[c + d*x])^n*Hypergeometric2F1[1/
2, 1/2 + n, 3/2 + n, Sin[(c + d*x)/2]^2]*Tan[(c + d*x)/2])/(d + 2*d*n)
```

Maple [F]

$$\int(a-\cos(dx+c)\,a)^n\,dx$$

```
[In] int((a-cos(d*x+c)*a)^n,x)
[Out] int((a-cos(d*x+c)*a)^n,x)
```

Fricas [F]

$$\int (a - a\cos(c + dx))^n \, dx = \int (-a\cos(dx + c) + a)^n \, dx$$

```
[In] integrate((a-a*cos(d*x+c))^n,x, algorithm="fricas")
[Out] integral((-a*cos(d*x + c) + a)^n, x)
```

Sympy [F]

$$\int (a - a\cos(c + dx))^n \, dx = \int (-a\cos(c + dx) + a)^n \, dx$$

```
[In] integrate((a-a*cos(d*x+c))**n,x)
[Out] Integral((-a*cos(c + d*x) + a)**n, x)
```

Maxima [F]

$$\int (a - a\cos(c + dx))^n \, dx = \int (-a\cos(dx + c) + a)^n \, dx$$

```
[In] integrate((a-a*cos(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((-a*cos(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (a - a\cos(c + dx))^n \, dx = \int (-a\cos(dx + c) + a)^n \, dx$$

```
[In] integrate((a-a*cos(d*x+c))^n,x, algorithm="giac")
[Out] integrate((-a*cos(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-a\cos(c+dx))^n\,dx=\int (a-a\,\cos(c+d\,x))^n\,dx$$

```
[In] int((a - a*cos(c + d*x))^n,x)
[Out] int((a - a*cos(c + d*x))^n, x)
```

3.16 $\int (2+2\cos(c+dx))^n\,dx$

Optimal result 163
Rubi [A] (verified) 163
Mathematica [A] (verified) 164
Maple [F] 164
Fricas [F] 164
Sympy [F] 165
Maxima [F] 165
Giac [F] 165
Mupad [F(-1)] 165

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int (2+2\cos(c+dx))^n\,dx = \frac{2^{\frac{1}{2}+2n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}}$$

```
[Out] 2^(1/2+2*n)*hypergeom([1/2, 1/2-n],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)/d/(
1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2730}

$$\int (2+2\cos(c+dx))^n\,dx = \frac{2^{2n+\frac{1}{2}}\sin(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{d\sqrt{\cos(c+dx)+1}}$$

```
[In] Int[(2 + 2*Cos[c + d*x])^n,x]
```

```
[Out] (2^(1/2 + 2*n)*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Cos[c + d*x])/2]*S
in[c + d*x])/(d*Sqrt[1 + Cos[c + d*x]])
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
```

```
b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{2^{\frac{1}{2}+2n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.31

$$\int (2+2\cos(c+dx))^n\,dx = -\frac{2^{1+n}(1+\cos(c+dx))^n\cot\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+n,\frac{3}{2}+n,\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\sin^2\left(\frac{1}{2}\right.}}{d+2dn}$$

```
[In] Integrate[(2 + 2*Cos[c + d*x])^n,x]
[Out] -((2^(1 + n)*(1 + Cos[c + d*x])^n*Cot[(c + d*x)/2]*Hypergeometric2F1[1/2, 1
/2 + n, 3/2 + n, Cos[(c + d*x)/2]^2]*Sqrt[Sin[(c + d*x)/2]^2])/(d + 2*d*n))
```

Maple [F]

$$\int (2+2\cos(dx+c))^n\,dx$$

```
[In] int((2+2*cos(d*x+c))^n,x)
[Out] int((2+2*cos(d*x+c))^n,x)
```

Fricas [F]

$$\int (2+2\cos(c+dx))^n\,dx = \int (2\cos(dx+c)+2)^n\,dx$$

```
[In] integrate((2+2*cos(d*x+c))^n,x, algorithm="fricas")
[Out] integral((2*cos(d*x + c) + 2)^n, x)
```

Sympy [F]

$$\int (2+2\cos(c+dx))^n\,dx = 2^n \int (\cos(c+dx)+1)^n\,dx$$

```
[In] integrate((2+2*cos(d*x+c))**n,x)
[Out] 2**n*Integral((cos(c + d*x) + 1)**n, x)
```

Maxima [F]

$$\int (2+2\cos(c+dx))^n\,dx = \int (2\cos(dx+c)+2)^n\,dx$$

```
[In] integrate((2+2*cos(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((2*cos(d*x + c) + 2)^n, x)
```

Giac [F]

$$\int (2+2\cos(c+dx))^n\,dx = \int (2\cos(dx+c)+2)^n\,dx$$

```
[In] integrate((2+2*cos(d*x+c))^n,x, algorithm="giac")
[Out] integrate((2*cos(d*x + c) + 2)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (2+2\cos(c+dx))^n\,dx = \int \left(4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)^n\,dx$$

```
[In] int((2*cos(c + d*x) + 2)^n,x)
[Out] int((4*cos(c/2 + (d*x)/2)^2)^n, x)
```

3.17 $\int (2-2\cos(c+dx))^n\,dx$

Optimal result . . . 166
Rubi [A] (verified) . . . 166
Mathematica [A] (verified) . . . 167
Maple [F] . . . 167
Fricas [F] . . . 167
Sympy [F] . . . 168
Maxima [F] . . . 168
Giac [F] . . . 168
Mupad [F(-1)] . . . 168

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int (2-2\cos(c+dx))^n\,dx = -\frac{2^{\frac{1}{2}+2n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1+\cos(c+dx))\right)\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}}$$

```
[Out] -2^(1/2+2*n)*hypergeom([1/2, 1/2-n],[3/2],1/2+1/2*cos(d*x+c))*sin(d*x+c)/d/
(1-cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2730}

$$\int (2-2\cos(c+dx))^n\,dx = -\frac{2^{2n+\frac{1}{2}}\sin(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(\cos(c+dx)+1)\right)}{d\sqrt{1-\cos(c+dx)}}$$

```
[In] Int[(2 - 2*Cos[c + d*x])^n,x]
```

```
[Out] -((2^(1/2 + 2*n)*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 + Cos[c + d*x])/2]
*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
```

```
b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = -\frac{2^{\frac{1}{2}+2n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1+\cos(c+dx))\right)\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int (2-2\cos(c+dx))^n\,dx = \frac{\sqrt{2}(2-2\cos(c+dx))^n\sqrt{1+\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+n,\frac{3}{2}+n,\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c\right.}{d+2dn}$$

```
[In] Integrate[(2 - 2*Cos[c + d*x])^n,x]
[Out] (Sqrt[2]*(2 - 2*Cos[c + d*x])^n*Sqrt[1 + Cos[c + d*x]]*Hypergeometric2F1[1/
2, 1/2 + n, 3/2 + n, Sin[(c + d*x)/2]^2]*Tan[(c + d*x)/2])/(d + 2*d*n)
```

Maple [F]

$$\int (-2\cos(dx+c)+2)^n\,dx$$

```
[In] int((-2*cos(d*x+c)+2)^n,x)
[Out] int((-2*cos(d*x+c)+2)^n,x)
```

Fricas [F]

$$\int (2-2\cos(c+dx))^n\,dx = \int (-2\cos(dx+c)+2)^n\,dx$$

```
[In] integrate((2-2*cos(d*x+c))^n,x, algorithm="fricas")
[Out] integral((-2*cos(d*x + c) + 2)^n, x)
```

Sympy [F]

$$\int (2-2\cos(c+dx))^n\,dx = \int (2-2\cos(c+dx))^n\ dx$$

```
[In] integrate((2-2*cos(d*x+c))**n,x)
[Out] Integral((2 - 2*cos(c + d*x))**n, x)
```

Maxima [F]

$$\int (2-2\cos(c+dx))^n\,dx = \int (-2\cos(dx+c)+2)^n\,dx$$

```
[In] integrate((2-2*cos(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((-2*cos(d*x + c) + 2)^n, x)
```

Giac [F]

$$\int (2-2\cos(c+dx))^n\,dx = \int (-2\cos(dx+c)+2)^n\,dx$$

```
[In] integrate((2-2*cos(d*x+c))^n,x, algorithm="giac")
[Out] integrate((-2*cos(d*x + c) + 2)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (2-2\cos(c+dx))^n\,dx = \int \left(4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)^n\,dx$$

```
[In] int((2 - 2*cos(c + d*x))^n,x)
[Out] int((4*sin(c/2 + (d*x)/2)^2)^n, x)
```

3.18 $\int \frac{1}{5+3\cos(c+dx)}\,dx$

Optimal result 169
Rubi [A] (verified) 169
Mathematica [A] (verified) 170
Maple [A] (verified) 170
Fricas [A] (verification not implemented) 170
Sympy [A] (verification not implemented) 171
Maxima [A] (verification not implemented) 171
Giac [A] (verification not implemented) 171
Mupad [B] (verification not implemented) 172

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{5+3\cos(c+dx)}\,dx = \frac{x}{4}-\frac{\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{2d}$$

```
[Out] 1/4*x-1/2*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2736}

$$\int \frac{1}{5+3\cos(c+dx)}\,dx = \frac{x}{4}-\frac{\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{2d}$$

```
[In] Int[(5 + 3*Cos[c + d*x])^(-1),x]
```

```
[Out] x/4 - ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])]/(2*d)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\text{integral} = \frac{x}{4}-\frac{\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{2d}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{1}{5+3\cos(c+dx)}\,dx = -\frac{\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

```
[In] Integrate[(5 + 3*Cos[c + d*x])^(-1),x]
[Out] -1/2*ArcTan[2*Cot[(c + d*x)/2]]/d
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{2d}$	18
default	$\frac{\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{2d}$	18
parallelrisch	$-\frac{i\left(\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2i\right)\right)}{4d}$	36
risch	$-\frac{i\ln\left(e^{i(dx+c)}+\frac{1}{3}\right)}{4d}+\frac{i\ln\left(e^{i(dx+c)}+3\right)}{4d}$	38

```
[In] int(1/(5+3*cos(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/2/d*arctan(1/2*tan(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{5+3\cos(c+dx)}\,dx = -\frac{\arctan\left(\frac{5\,\cos(dx+c)+3}{4\,\sin(dx+c)}\right)}{4\,d}$$

```
[In] integrate(1/(5+3*cos(d*x+c)),x, algorithm="fricas")
[Out] -1/4*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{1}{5+3\cos(c+dx)}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor}{2d} & \text{for } d\neq 0 \\ \frac{x}{3\cos{(c)}+5} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(5+3*cos(d*x+c)),x)
[Out] Piecewise(((atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(
2*d), Ne(d, 0)), (x/(3*cos(c) + 5), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{5+3\cos(c+dx)}\,dx = \frac{\arctan\left(\frac{\sin(dx+c)}{2\,(\cos(dx+c)+1)}\right)}{2\,d}$$

```
[In] integrate(1/(5+3*cos(d*x+c)),x, algorithm="maxima")
[Out] 1/2*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{5+3\cos(c+dx)}\,dx = \frac{dx+c-2\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{4\,d}$$

```
[In] integrate(1/(5+3*cos(d*x+c)),x, algorithm="giac")
[Out] 1/4*(d*x + c - 2*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{1}{5+3\cos(c+dx)}\,dx=\frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{2\,d}-\frac{\operatorname{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}}{2\,d}$$

```
[In] int(1/(3*cos(c + d*x) + 5),x)
[Out] atan(tan(c/2 + (d*x)/2)/2)/(2*d) - (atan(tan(c/2 + (d*x)/2)) - (d*x)/2)/(2*
d)
```

3.19 $\int \frac{1}{(5+3\cos(c+dx))^2}\,dx$

Optimal result . . . 173
Rubi [A] (verified) . . . 173
Mathematica [A] (verified) . . . 174
Maple [A] (verified) . . . 174
Fricas [A] (verification not implemented) . . . 175
Sympy [C] (verification not implemented) . . . 175
Maxima [A] (verification not implemented) . . . 176
Giac [A] (verification not implemented) . . . 176
Mupad [B] (verification not implemented) . . . 177

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(5+3\cos(c+dx))^2}\,dx = \frac{5x}{64} - \frac{5\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{32d} - \frac{3\sin(c+dx)}{16d(5+3\cos(c+dx))}$$

```
[Out] 5/64*x-5/32*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-3/16*sin(d*x+c)/d/(5+3*cos(
d*x+c))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2743, 12, 2736}

$$\int \frac{1}{(5+3\cos(c+dx))^2}\,dx = -\frac{5\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{32d} - \frac{3\sin(c+dx)}{16d(3\cos(c+dx)+5)} + \frac{5x}{64}$$

```
[In] Int[(5 + 3*Cos[c + d*x])^(-2),x]
```

```
[Out] (5*x)/64 - (5*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(32*d) - (3*Sin[c +
d*x])/(16*d*(5 + 3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3\sin(c+dx)}{16d(5+3\cos(c+dx))} - \frac{1}{16}\int -\frac{5}{5+3\cos(c+dx)}\,dx \\
&= -\frac{3\sin(c+dx)}{16d(5+3\cos(c+dx))} + \frac{5}{16}\int \frac{1}{5+3\cos(c+dx)}\,dx \\
&= \frac{5x}{64} - \frac{5\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{32d} - \frac{3\sin(c+dx)}{16d(5+3\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{1}{(5+3\cos(c+dx))^2}\,dx = -\frac{5\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right) + \frac{6\sin(c+dx)}{5+3\cos(c+dx)}}{32d}$$

```
[In] Integrate[(5 + 3*Cos[c + d*x])^(-2),x]
[Out] -1/32*(5*ArcTan[2*Cot[(c + d*x)/2]] + (6*Sin[c + d*x])/(5 + 3*Cos[c + d*x])
)/d
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)}+\frac{5\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{32}}{d}$	46
default	$\frac{-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)}+\frac{5\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{32}}{d}$	46
parallelrisch	$\frac{(-15i\cos(dx+c)-25i)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+(15i\cos(dx+c)+25i)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2i\right)-12\sin(dx+c)}{192d\cos(dx+c)+320d}$	78
risch	$-\frac{i\left(5\,\mathrm{e}^{i(dx+c)}+3\right)}{8d\left(3\,\mathrm{e}^{2i(dx+c)}+10\,\mathrm{e}^{i(dx+c)}+3\right)}-\frac{5i\ln\left(\mathrm{e}^{i(dx+c)}+\frac{1}{3}\right)}{64d}+\frac{5i\ln\left(\mathrm{e}^{i(dx+c)}+3\right)}{64d}$	83

```
[In] int(1/(5+3*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(-3/16*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+4)+5/32*arctan(1/2*tan(
1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05

$$\int \frac{1}{(5+3\cos(c+dx))^2}\,dx = -\frac{5\left(3\cos\left(dx+c\right)+5\right)\arctan\left(\frac{5\cos(dx+c)+3}{4\sin(dx+c)}\right)+12\sin\left(dx+c\right)}{64\left(3\,d\cos\left(dx+c\right)+5\,d\right)}$$

```
[In] integrate(1/(5+3*cos(d*x+c))^2,x, algorithm="fricas")
[Out] -1/64*(5*(3*cos(d*x + c) + 5)*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c))
 + 12*sin(d*x + c))/(3*d*cos(d*x + c) + 5*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.34

$$\int \frac{1}{(5+3\cos(c+dx))^2}\,dx = \begin{cases} \frac{x}{(5+3\cosh\left(2\,\mathrm{atanh}\left(2\right)\right))^2} & \text{for} \\ \frac{x}{(3\cos\left(c\right)+5)^2} & \text{for} \\ \frac{5\left(\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{32d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+128d}+\frac{20\left(\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{32d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+128d}-\frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{32d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+128d} & \text{oth} \end{cases}$$

```
[In] integrate(1/(5+3*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((x/(5 + 3*cosh(2*atanh(2)))**2, Eq(c, -d*x - 2*I*atanh(2)) | Eq(c
, -d*x + 2*I*atanh(2))), (x/(3*cos(c) + 5)**2, Eq(d, 0)), (5*(atan(tan(c/2
+ d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(32*d*
tan(c/2 + d*x/2)**2 + 128*d) + 20*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2
 + d*x/2 - pi/2)/pi))/(32*d*tan(c/2 + d*x/2)**2 + 128*d) - 6*tan(c/2 + d*x/
2)/(32*d*tan(c/2 + d*x/2)**2 + 128*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{1}{(5+3\cos(c+dx))^2}\,dx = -\frac{\frac{6\,\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+4\right)(\cos(dx+c)+1)} - 5\,\arctan\left(\frac{\sin(dx+c)}{2\,(\cos(dx+c)+1)}\right)}{32\,d}$$

```
[In] integrate(1/(5+3*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/32*(6*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4)*(cos(d*x +
 c) + 1)) - 5*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05

$$\int \frac{1}{(5+3\cos(c+dx))^2}\,dx = \frac{5\,dx + 5\,c - \frac{12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4} - 10\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{64\,d}$$

```
[In] integrate(1/(5+3*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(5*d*x + 5*c - 12*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 4) -
10*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

Mupad [B] (verification not implemented)

Time = 15.52 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

$$\int \frac{1}{(5+3\cos(c+dx))^2}\,dx = \frac{5\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{32\,d} - \frac{5\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{32\,d} - \frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\right)}$$

```
[In] int(1/(3*cos(c + d*x) + 5)^2,x)
[Out] (5*atan(tan(c/2 + (d*x)/2)/2))/(32*d) - (5*(atan(tan(c/2 + (d*x)/2)) - (d*x
)/2))/(32*d) - (3*tan(c/2 + (d*x)/2))/(16*d*(tan(c/2 + (d*x)/2)^2 + 4))
```

3.20 $\int \frac{1}{(5+3\cos(c+dx))^3}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 81

$$\int \frac{1}{(5+3\cos(c+dx))^3}\,dx = \frac{59x}{2048}-\frac{59\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{1024d}-\frac{3\sin(c+dx)}{32d(5+3\cos(c+dx))^2}-\frac{45\sin(c+dx)}{512d(5+3\cos(c+dx))}$$

```
[Out] 59/2048*x-59/1024*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-3/32*sin(d*x+c)/d/(5+
3*cos(d*x+c))^2-45/512*sin(d*x+c)/d/(5+3*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2743, 2833, 12, 2736}

$$\int \frac{1}{(5+3\cos(c+dx))^3}\,dx = -\frac{59\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{1024d}-\frac{45\sin(c+dx)}{512d(3\cos(c+dx)+5)}-\frac{3\sin(c+dx)}{32d(3\cos(c+dx)+5)^2}+\frac{59x}{2048}$$

```
[In] Int[(5 + 3*Cos[c + d*x])^(-3),x]
```

```
[Out] (59*x)/2048 - (59*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(1024*d) - (3*Si
n[c + d*x])/(32*d*(5 + 3*Cos[c + d*x])^2) - (45*Sin[c + d*x])/(512*d*(5 + 3
*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3\sin(c+dx)}{32d(5+3\cos(c+dx))^2}-\frac{1}{32}\int\frac{-10+3\cos(c+dx)}{(5+3\cos(c+dx))^2}\,dx\\
&= -\frac{3\sin(c+dx)}{32d(5+3\cos(c+dx))^2}-\frac{45\sin(c+dx)}{512d(5+3\cos(c+dx))}+\frac{1}{512}\int\frac{59}{5+3\cos(c+dx)}\,dx\\
&= -\frac{3\sin(c+dx)}{32d(5+3\cos(c+dx))^2}-\frac{45\sin(c+dx)}{512d(5+3\cos(c+dx))}+\frac{59}{512}\int\frac{1}{5+3\cos(c+dx)}\,dx\\
&= \frac{59x}{2048}-\frac{59\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{1024d}-\frac{3\sin(c+dx)}{32d(5+3\cos(c+dx))^2}-\frac{45\sin(c+dx)}{512d(5+3\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

$$\int \frac{1}{(5+3\cos(c+dx))^3}\,dx = -\frac{59\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)+\frac{3(182\sin(c+dx)+45\sin(2(c+dx)))}{(5+3\cos(c+dx))^2}}{1024d}$$

```
[In] Integrate[(5 + 3*Cos[c + d*x])^(-3),x]
```

```
[Out] -1/1024*(59*ArcTan[2*Cot[(c + d*x)/2]] + (3*(182*Sin[c + d*x] + 45*Sin[2*(c
 + d*x)]))/(5 + 3*Cos[c + d*x])^2)/d
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{\frac{-\frac{69\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}}{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^2}+\frac{59\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1024}}{d}$
default	$\frac{\frac{-\frac{69\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}}{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^2}+\frac{59\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1024}}{d}$
risch	$-\frac{3i\left(59\,e^{3i(dx+c)}+295\,e^{2i(dx+c)}+241\,e^{i(dx+c)}+45\right)}{256d\left(3\,e^{2i(dx+c)}+10\,e^{i(dx+c)}+3\right)^2}-\frac{59i\ln\left(e^{i(dx+c)}+\frac{1}{3}\right)}{2048d}+\frac{59i\ln\left(e^{i(dx+c)}+3\right)}{2048d}$
parallelrisch	$\frac{59i(-59-9\cos(2dx+2c)-60\cos(dx+c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+59i(59+9\cos(2dx+2c)+60\cos(dx+c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right.}{2048d(59+9\cos(2dx+2c)+60\cos(dx+c))}$

```
[In] int(1/(5+3*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*(-69/128*tan(1/2*d*x+1/2*c)^3-51/32*tan(1/2*d*x+1/2*c))/(tan(1/2*d
*x+1/2*c)^2+4)^2+59/1024*arctan(1/2*tan(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \frac{1}{(5+3\cos(c+dx))^3}\,dx =$$

$$-\frac{59\left(9\cos(dx+c)^2+30\cos(dx+c)+25\right)\arctan\left(\frac{5\cos(dx+c)+3}{4\sin(dx+c)}\right)+12\left(45\cos(dx+c)+91\right)\sin(dx+c}{2048\left(9\,d\cos(dx+c)^2+30\,d\cos(dx+c)+25\,d\right)}$$

```
[In] integrate(1/(5+3*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2048*(59*(9*cos(d*x + c)^2 + 30*cos(d*x + c) + 25)*arctan(1/4*(5*cos(d*x
 + c) + 3)/sin(d*x + c)) + 12*(45*cos(d*x + c) + 91)*sin(d*x + c))/(9*d*cos
(d*x + c)^2 + 30*d*cos(d*x + c) + 25*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 359, normalized size of antiderivative = 4.43

$$\int \frac{1}{(5+3\cos(c+dx))^3}\,dx = \begin{cases} \frac{x}{(5+3\cosh(2\operatorname{atanh}(2)))^3} \\ \frac{x}{(3\cos(c)+5)^3} \\ \frac{59\left(\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{1024d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+16384d} + \frac{472\left(\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1024d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+16384d} + \frac{944\left(\operatorname{atan}\left(\right.\right.}{1024d\tan^4\left(\frac{c}{2}\right.} \end{cases}$$

```
[In] integrate(1/(5+3*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((x/(5 + 3*cosh(2*atanh(2)))**3, Eq(c, -d*x - 2*I*atanh(2)) | Eq(c
, -d*x + 2*I*atanh(2))), (x/(3*cos(c) + 5)**3, Eq(d, 0)), (59*(atan(tan(c/2
 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(1024
*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d) + 472*(atan(
tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**
2/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d) + 944
*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(1024*d*tan
(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d) - 138*tan(c/2 + d*
x/2)**3/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d)
 - 408*tan(c/2 + d*x/2)/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/
2)**2 + 16384*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

$$\int \frac{1}{(5+3\cos(c+dx))^3}\,dx = -\frac{\frac{6\left(\frac{68\sin(dx+c)}{\cos(dx+c)+1}+\frac{23\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+16} - 59\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{1024\,d}$$

```
[In] integrate(1/(5+3*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/1024*(6*(68*sin(d*x + c)/(cos(d*x + c) + 1) + 23*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 + 16) - 59*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{1}{(5+3\cos(c+dx))^3}\,dx = \frac{59\,dx+59\,c-\frac{12\left(23\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+68\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\right)^2}-118\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{2048\,d}$$

```
[In] integrate(1/(5+3*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2048*(59*d*x + 59*c - 12*(23*tan(1/2*d*x + 1/2*c)^3 + 68*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 + 4)^2 - 118*arctan(sin(d*x + c)/(cos(d*x + c
) + 3)))/d
```

Mupad [B] (verification not implemented)

Time = 15.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19

$$\int \frac{1}{(5+3\cos(c+dx))^3}\,dx = \frac{59\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{1024\,d} - \frac{59\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{1024\,d} - \frac{\frac{69\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{512}+\frac{51\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+16\right)}$$

```
[In] int(1/(3*cos(c + d*x) + 5)^3,x)
```

```
[Out] (59*atan(tan(c/2 + (d*x)/2)/2))/(1024*d) - (59*(atan(tan(c/2 + (d*x)/2)) -
(d*x)/2))/(1024*d) - ((51*tan(c/2 + (d*x)/2))/128 + (69*tan(c/2 + (d*x)/2)^
3)/512)/(d*(8*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 16))
```

3.21 $\int \frac{1}{(5+3\cos(c+dx))^4}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 106

$$\int \frac{1}{(5+3\cos(c+dx))^4}\,dx = \frac{385x}{32768} - \frac{385\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{16384d} - \frac{\sin(c+dx)}{16d(5+3\cos(c+dx))^3} - \frac{25\sin(c+dx)}{512d(5+3\cos(c+dx))^2} - \frac{311\sin(c+dx)}{8192d(5+3\cos(c+dx))}$$

```
[Out] 385/32768*x-385/16384*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-1/16*sin(d*x+c)/d
/(5+3*cos(d*x+c))^3-25/512*sin(d*x+c)/d/(5+3*cos(d*x+c))^2-311/8192*sin(d*x
+c)/d/(5+3*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2743, 2833, 12, 2736}

$$\int \frac{1}{(5+3\cos(c+dx))^4}\,dx = -\frac{385\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{16384d} - \frac{311\sin(c+dx)}{8192d(3\cos(c+dx)+5)} - \frac{25\sin(c+dx)}{512d(3\cos(c+dx)+5)^2} - \frac{\sin(c+dx)}{16d(3\cos(c+dx)+5)^3} + \frac{385x}{32768}$$

```
[In] Int[(5 + 3*Cos[c + d*x])^(-4),x]
```

```
[Out] (385*x)/32768 - (385*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(16384*d) - S
in[c + d*x]/(16*d*(5 + 3*Cos[c + d*x])^3) - (25*Sin[c + d*x])/(512*d*(5 + 3
*Cos[c + d*x])^2) - (311*Sin[c + d*x])/(8192*d*(5 + 3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)}{16d(5+3\cos(c+dx))^3} - \frac{1}{48}\int \frac{-15+6\cos(c+dx)}{(5+3\cos(c+dx))^3}\,dx \\
&= -\frac{\sin(c+dx)}{16d(5+3\cos(c+dx))^3} - \frac{25\sin(c+dx)}{512d(5+3\cos(c+dx))^2} + \frac{\int \frac{186-75\cos(c+dx)}{(5+3\cos(c+dx))^2}\,dx}{1536} \\
&= -\frac{\sin(c+dx)}{16d(5+3\cos(c+dx))^3} - \frac{25\sin(c+dx)}{512d(5+3\cos(c+dx))^2} \\
&\quad - \frac{311\sin(c+dx)}{8192d(5+3\cos(c+dx))} - \frac{\int -\frac{1155}{5+3\cos(c+dx)}\,dx}{24576} \\
&= -\frac{\sin(c+dx)}{16d(5+3\cos(c+dx))^3} - \frac{25\sin(c+dx)}{512d(5+3\cos(c+dx))^2} \\
&\quad - \frac{311\sin(c+dx)}{8192d(5+3\cos(c+dx))} + \frac{385\int \frac{1}{5+3\cos(c+dx)}\,dx}{8192}
\end{aligned}
$$

$$= \frac{385x}{32768} - \frac{385\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{16384d} - \frac{\sin(c+dx)}{16d(5+3\cos(c+dx))^3} - \frac{25\sin(c+dx)}{512d(5+3\cos(c+dx))^2} - \frac{311\sin(c+dx)}{8192d(5+3\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int \frac{1}{(5+3\cos(c+dx))^4}\,dx = -\frac{770\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right) + \frac{9(4883\sin(c+dx)+2340\sin(2(c+dx))+311\sin(3(c+dx)))}{(5+3\cos(c+dx))^3}}{32768d}$$

```
[In] Integrate[(5 + 3*Cos[c + d*x])^(-4),x]
```

```
[Out] -1/32768*(770*ArcTan[2*Cot[(c + d*x)/2]] + (9*(4883*Sin[c + d*x] + 2340*Sin
[2*(c + d*x)] + 311*Sin[3*(c + d*x)]))/(5 + 3*Cos[c + d*x])^3)/d
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{\frac{-\frac{639\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}-\frac{117\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{369\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}}{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^3}+\frac{385\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{16384}}{d}$
default	$\frac{\frac{-\frac{639\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}-\frac{117\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{369\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}}{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^3}+\frac{385\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{16384}}{d}$
risch	$-\frac{i\left(10395\,\mathrm{e}^{5i(dx+c)}+86625\,\mathrm{e}^{4i(dx+c)}+239470\,\mathrm{e}^{3i(dx+c)}+218466\,\mathrm{e}^{2i(dx+c)}+73575\,\mathrm{e}^{i(dx+c)}+8397\right)}{12288d\left(3\,\mathrm{e}^{2i(dx+c)}+10\,\mathrm{e}^{i(dx+c)}+3\right)^3}+\frac{385i\ln(\mathrm{e}^{i(dx+c)}}{32768d}$
parallelrisch	$\frac{385i(-770-27\cos(3dx+3c)-981\cos(dx+c)-270\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+385i(770+27\cos(3dx+3c)+981\,c}{32768d(770+27\cos(3dx+3c)+981\,c}$

```
[In] int(1/(5+3*cos(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*(-639/1024*tan(1/2*d*x+1/2*c)^5-117/32*tan(1/2*d*x+1/2*c)^3-369/64
*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2+4)^3+385/16384*arctan(1/2*tan(1/
2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14

$$\int \frac{1}{(5+3\cos(c+dx))^4}\,dx = -\frac{385\left(27\,\cos\left(dx+c\right)^3+135\,\cos\left(dx+c\right)^2+225\,\cos\left(dx+c\right)+125\right)\arctan\left(\frac{5\,\cos(dx+c)+3}{4\,\sin(dx+c)}\right)+36\left(311\,\cos\right.}{32768\left(27\,d\cos\left(dx+c\right)^3+135\,d\cos\left(dx+c\right)^2+225\,d\cos\left(dx+\right.\right.}$$

```
[In] integrate(1/(5+3*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/32768*(385*(27*cos(d*x + c)^3 + 135*cos(d*x + c)^2 + 225*cos(d*x + c) +
125)*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c)) + 36*(311*cos(d*x + c)^2
 + 1170*cos(d*x + c) + 1143)*sin(d*x + c))/(27*d*cos(d*x + c)^3 + 135*d*cos
(d*x + c)^2 + 225*d*cos(d*x + c) + 125*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.07 (sec) , antiderivative size = 592, normalized size of antiderivative = 5.58

$$\int \frac{1}{(5+3\cos(c+dx))^4}\,dx = \begin{cases} \frac{x}{(5+3\cosh(2\,\mathrm{atanh}\,(2)))^4} \\ \frac{x}{(3\cos(c)+5)^4} \\ \frac{385\left(\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{16384d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+196608d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+786432d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1048576d} + \frac{4620\left(\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{16384d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+196608d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+786432d} \end{cases}$$

```
[In] integrate(1/(5+3*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((x/(5 + 3*cosh(2*atanh(2)))**4, Eq(c, -d*x - 2*I*atanh(2)) | Eq(c
, -d*x + 2*I*atanh(2))), (x/(3*cos(c) + 5)**4, Eq(d, 0)), (385*(atan(tan(c/
2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**6/(163
84*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2
+ d*x/2)**2 + 1048576*d) + 4620*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 +
 d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(16384*d*tan(c/2 + d*x/2)**6 + 1966
08*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) + 1848
0*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 +
d*x/2)**2/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786
432*d*tan(c/2 + d*x/2)**2 + 1048576*d) + 24640*(atan(tan(c/2 + d*x/2)/2) +
```

```
pi*floor((c/2 + d*x/2 - pi/2)/pi))/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*
tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) - 1278*tan(
c/2 + d*x/2)**5/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4
 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) - 7488*tan(c/2 + d*x/2)**3/(16
384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2
 + d*x/2)**2 + 1048576*d) - 11808*tan(c/2 + d*x/2)/(16384*d*tan(c/2 + d*x/2
)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 104857
6*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.42

$$\int \frac{1}{(5+3\cos(c+dx))^4}\,dx = -\frac{\frac{18\left(\frac{656\sin(dx+c)}{\cos(dx+c)+1}+\frac{416\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{71\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{12\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+64}-385\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{16384\,d}$$

```
[In] integrate(1/(5+3*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/16384*(18*(656*sin(d*x + c)/(cos(d*x + c) + 1) + 416*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 + 71*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 + 64) - 385*arctan(1/2*sin(d*x + c)/(cos(d*x +
c) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{1}{(5+3\cos(c+dx))^4}\,dx = \frac{385\,dx+385\,c-\frac{36\left(71\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+416\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+656\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\right)^3}-770\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{32768\,d}$$

```
[In] integrate(1/(5+3*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/32768*(385*d*x + 385*c - 36*(71*tan(1/2*d*x + 1/2*c)^5 + 416*tan(1/2*d*x
+ 1/2*c)^3 + 656*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 4)^3 - 770
*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

Mupad [B] (verification not implemented)

Time = 15.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{1}{(5+3\cos(c+dx))^4}\,dx = \frac{385\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{16384\,d} - \frac{385\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{16384\,d} - \frac{\frac{639\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8192}+\frac{117\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{256}+\frac{369\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{512}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\right)^3}$$

```
[In] int(1/(3*cos(c + d*x) + 5)^4,x)
```

```
[Out] (385*atan(tan(c/2 + (d*x)/2)/2))/(16384*d) - (385*(atan(tan(c/2 + (d*x)/2))
 - (d*x)/2))/(16384*d) - ((369*tan(c/2 + (d*x)/2))/512 + (117*tan(c/2 + (d*
x)/2)^3)/256 + (639*tan(c/2 + (d*x)/2)^5)/8192)/(d*(tan(c/2 + (d*x)/2)^2 +
4)^3)
```

3.22 $\int \frac{1}{5-3\cos(c+dx)}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{1}{5-3\cos(c+dx)}\,dx=\frac{x}{4}+\frac{\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{2d}$$

```
[Out] 1/4*x+1/2*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2736}

$$\int \frac{1}{5-3\cos(c+dx)}\,dx=\frac{\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{2d}+\frac{x}{4}$$

```
[In] Int[(5 - 3*Cos[c + d*x])^(-1),x]
[Out] x/4 + ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])]/(2*d)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\text{integral}=\frac{x}{4}+\frac{\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{2d}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

$$\int \frac{1}{5-3\cos(c+dx)}\,dx = \frac{\arctan\left(2\tan\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

```
[In] Integrate[(5 - 3*Cos[c + d*x])^(-1),x]
[Out] ArcTan[2*Tan[(c + d*x)/2]]/(2*d)
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.55

method	result	size
derivativedivides	$\frac{\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}$	18
default	$\frac{\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}$	18
risch	$-\frac{i\ln\left(e^{i(dx+c)}-\frac{1}{3}\right)}{4d}+\frac{i\ln\left(e^{i(dx+c)}-3\right)}{4d}$	38
parallelrisch	$-\frac{i\left(\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)-\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)\right)}{4d}$	40

```
[In] int(1/(5-3*cos(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/2/d*arctan(2*tan(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{5-3\cos(c+dx)}\,dx = -\frac{\arctan\left(\frac{5\cos(dx+c)-3}{4\sin(dx+c)}\right)}{4\,d}$$

```
[In] integrate(1/(5-3*cos(d*x+c)),x, algorithm="fricas")
[Out] -1/4*arctan(1/4*(5*cos(d*x + c) - 3)/sin(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{1}{5-3\cos(c+dx)}\,dx = \begin{cases} \frac{\operatorname{atan}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor}{2d} & \text{for } d\neq 0 \\ \frac{x}{5-3\cos(c)} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(5-3*cos(d*x+c)),x)
[Out] Piecewise(((atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(
2*d), Ne(d, 0)), (x/(5 - 3*cos(c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{1}{5-3\cos(c+dx)}\,dx = \frac{\arctan\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}\right)}{2d}$$

```
[In] integrate(1/(5-3*cos(d*x+c)),x, algorithm="maxima")
[Out] 1/2*arctan(2*sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{1}{5-3\cos(c+dx)}\,dx = \frac{dx+c-2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{4d}$$

```
[In] integrate(1/(5-3*cos(d*x+c)),x, algorithm="giac")
[Out] 1/4*(d*x + c - 2*arctan(sin(d*x + c)/(cos(d*x + c) - 3)))/d
```

Mupad [B] (verification not implemented)

Time = 15.49 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{1}{5-3\cos(c+dx)}\,dx=\frac{\operatorname{atan}\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{\operatorname{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}}{2\,d}$$

```
[In] int(-1/(3*cos(c + d*x) - 5),x)
[Out] atan(2*tan(c/2 + (d*x)/2))/(2*d) - (atan(tan(c/2 + (d*x)/2)) - (d*x)/2)/(2*
d)
```

3.23 $\int \frac{1}{(5-3\cos(c+dx))^2}\,dx$

Optimal result . . . 193
Rubi [A] (verified) . . . 193
Mathematica [A] (verified) . . . 194
Maple [A] (verified) . . . 194
Fricas [A] (verification not implemented) . . . 195
Sympy [C] (verification not implemented) . . . 195
Maxima [A] (verification not implemented) . . . 196
Giac [A] (verification not implemented) . . . 196
Mupad [B] (verification not implemented) . . . 197

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{(5-3\cos(c+dx))^2}\,dx = \frac{5x}{64} + \frac{5\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{32d} + \frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))}$$

```
[Out] 5/64*x+5/32*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d+3/16*sin(d*x+c)/d/(5-3*cos(
d*x+c))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2743, 12, 2736}

$$\int \frac{1}{(5-3\cos(c+dx))^2}\,dx = \frac{5\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{32d} + \frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))} + \frac{5x}{64}$$

```
[In] Int[(5 - 3*Cos[c + d*x])^(-2),x]
```

```
[Out] (5*x)/64 + (5*ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])])/(32*d) + (3*Sin[c +
d*x])/(16*d*(5 - 3*Cos[c + d*x]))
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 2736</u>

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))} - \frac{1}{16}\int -\frac{5}{5-3\cos(c+dx)}\,dx \\
&= \frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))} + \frac{5}{16}\int \frac{1}{5-3\cos(c+dx)}\,dx \\
&= \frac{5x}{64} + \frac{5\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{32d} + \frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int \frac{1}{(5-3\cos(c+dx))^2}\,dx = \frac{5\arctan\left(2\tan\left(\frac{1}{2}(c+dx)\right)\right) - \frac{6\sin(c+dx)}{-5+3\cos(c+dx)}}{32d}$$

```
[In] Integrate[(5 - 3*Cos[c + d*x])^(-2),x]
[Out] (5*ArcTan[2*Tan[(c + d*x)/2]] - (6*Sin[c + d*x])/(-5 + 3*Cos[c + d*x]))/(32
*d)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{4}\right)}+\frac{5\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}}{d}$	46
default	$\frac{\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{4}\right)}+\frac{5\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}}{d}$	46
parallelrisch	$\frac{(-15i\cos(dx+c)+25i)\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+(15i\cos(dx+c)-25i)\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)-12\sin(dx+c)}{192d\cos(dx+c)-320d}$	82
risch	$\frac{i\left(5\,e^{i(dx+c)}-3\right)}{8d\left(3\,e^{2i(dx+c)}-10\,e^{i(dx+c)}+3\right)}+\frac{5i\ln\left(e^{i(dx+c)}-3\right)}{64d}-\frac{5i\ln\left(e^{i(dx+c)}-\frac{1}{3}\right)}{64d}$	83

```
[In] int(1/(5-3*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(3/64*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1/4)+5/32*arctan(2*tan(1
/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{1}{(5-3\cos(c+dx))^2}\,dx = -\frac{5\left(3\,\cos\left(dx+c\right)-5\right)\arctan\left(\frac{5\,\cos(dx+c)-3}{4\,\sin(dx+c)}\right)+12\,\sin\left(dx+c\right)}{64\left(3\,d\cos\left(dx+c\right)-5\,d\right)}$$

```
[In] integrate(1/(5-3*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/64*(5*(3*cos(d*x + c) - 5)*arctan(1/4*(5*cos(d*x + c) - 3)/sin(d*x + c))
 + 12*sin(d*x + c))/(3*d*cos(d*x + c) - 5*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.31

$$\int \frac{1}{(5-3\cos(c+dx))^2}\,dx = \begin{cases} \frac{x}{\left(5-3\cosh\left(2\,\text{atanh}\left(\frac{1}{2}\right)\right)\right)^2} & \text{fo} \\ \frac{x}{\left(5-3\cos\left(c\right)\right)^2} & \text{fo} \\ \frac{20\left(\text{atan}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{128d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+32d}+\frac{5\left(\text{atan}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{128d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+32d}+\frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+32d} & \text{o} \end{cases}$$

```
[In] integrate(1/(5-3*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((x/(5 - 3*cosh(2*atanh(1/2)))**2, Eq(c, -d*x - 2*I*atanh(1/2)) |
Eq(c, -d*x + 2*I*atanh(1/2))), (x/(5 - 3*cos(c))**2, Eq(d, 0)), (20*(atan(2
*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2
/(128*d*tan(c/2 + d*x/2)**2 + 32*d) + 5*(atan(2*tan(c/2 + d*x/2)) + pi*floo
r((c/2 + d*x/2 - pi/2)/pi))/(128*d*tan(c/2 + d*x/2)**2 + 32*d) + 6*tan(c/2
+ d*x/2)/(128*d*tan(c/2 + d*x/2)**2 + 32*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int \frac{1}{(5-3\cos(c+dx))^2}\,dx = \frac{\frac{6\,\sin(dx+c)}{\left(\frac{4\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)} + 5\,\arctan\left(\frac{2\,\sin(dx+c)}{\cos(dx+c)+1}\right)}{32\,d}$$

```
[In] integrate(1/(5-3*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/32*(6*sin(d*x + c)/((4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x
+ c) + 1)) + 5*arctan(2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int \frac{1}{(5-3\cos(c+dx))^2}\,dx = \frac{5\,dx + 5\,c + \frac{12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} - 10\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{64\,d}$$

```
[In] integrate(1/(5-3*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(5*d*x + 5*c + 12*tan(1/2*d*x + 1/2*c)/(4*tan(1/2*d*x + 1/2*c)^2 + 1)
- 10*arctan(sin(d*x + c)/(cos(d*x + c) - 3)))/d
```

Mupad [B] (verification not implemented)

Time = 15.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \frac{1}{(5-3\cos(c+dx))^2}\,dx = \frac{5\,\mathrm{atan}\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{32\,d} - \frac{5\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{32\,d} + \frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{1}{4}\right)}$$

```
[In] int(1/(3*cos(c + d*x) - 5)^2,x)
```

```
[Out] (5*atan(2*tan(c/2 + (d*x)/2)))/(32*d) - (5*(atan(tan(c/2 + (d*x)/2)) - (d*x
)/2))/(32*d) + (3*tan(c/2 + (d*x)/2))/(64*d*(tan(c/2 + (d*x)/2)^2 + 1/4))
```

3.24 $\int \frac{1}{(5-3\cos(c+dx))^3}\,dx$

Optimal result 198
Rubi [A] (verified) 198
Mathematica [A] (verified) 200
Maple [A] (verified) 200
Fricas [A] (verification not implemented) 200
Sympy [C] (verification not implemented) 201
Maxima [A] (verification not implemented) 201
Giac [A] (verification not implemented) 202
Mupad [B] (verification not implemented) 202

Optimal result

Integrand size = 12, antiderivative size = 83

$$\int \frac{1}{(5-3\cos(c+dx))^3}\,dx = \frac{59x}{2048}+\frac{59\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{1024d}+\frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2}+\frac{45\sin(c+dx)}{512d(5-3\cos(c+dx))}$$

```
[Out] 59/2048*x+59/1024*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d+3/32*sin(d*x+c)/d/(5-
3*cos(d*x+c))^2+45/512*sin(d*x+c)/d/(5-3*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2743, 2833, 12, 2736}

$$\int \frac{1}{(5-3\cos(c+dx))^3}\,dx = \frac{59\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{1024d}+\frac{45\sin(c+dx)}{512d(5-3\cos(c+dx))}+\frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2}+\frac{59x}{2048}$$

```
[In] Int[(5 - 3*Cos[c + d*x])^(-3),x]
```

```
[Out] (59*x)/2048 + (59*ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])])/(1024*d) + (3*Si
n[c + d*x])/(32*d*(5 - 3*Cos[c + d*x])^2) + (45*Sin[c + d*x])/(512*d*(5 - 3
*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2} - \frac{1}{32}\int \frac{-10-3\cos(c+dx)}{(5-3\cos(c+dx))^2}\,dx \\
&= \frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(5-3\cos(c+dx))} + \frac{1}{512}\int \frac{59}{5-3\cos(c+dx)}\,dx \\
&= \frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(5-3\cos(c+dx))} + \frac{59}{512}\int \frac{1}{5-3\cos(c+dx)}\,dx \\
&= \frac{59x}{2048} + \frac{59\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{1024d} + \frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(5-3\cos(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{1}{(5-3\cos(c+dx))^3}\,dx = \frac{59\arctan\left(2\tan\left(\frac{1}{2}(c+dx)\right)\right)(5-3\cos(c+dx))^2+546\sin(c+dx)-135\sin(2(c+dx))}{1024d(5-3\cos(c+dx))^2}$$

```
[In] Integrate[(5 - 3*Cos[c + d*x])^(-3),x]
[Out] (59*ArcTan[2*Tan[(c + d*x)/2]]*(5 - 3*Cos[c + d*x])^2 + 546*Sin[c + d*x] -
135*Sin[2*(c + d*x)])/(1024*d*(5 - 3*Cos[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{\frac{\frac{51\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}+\frac{69\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512}}{\left(4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)^2}+\frac{59\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}}{d}$
default	$\frac{\frac{\frac{51\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}+\frac{69\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512}}{\left(4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)^2}+\frac{59\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}}{d}$
risch	$\frac{3i\left(59\,e^{3i(dx+c)}-295\,e^{2i(dx+c)}+241\,e^{i(dx+c)}-45\right)}{256d\left(3\,e^{2i(dx+c)}-10\,e^{i(dx+c)}+3\right)^2}-\frac{59i\ln\left(e^{i(dx+c)}-\frac{1}{3}\right)}{2048d}+\frac{59i\ln\left(e^{i(dx+c)}-3\right)}{2048d}$
parallelrisch	$\frac{59i(59+9\cos(2dx+2c)-60\cos(dx+c))\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+59i(-9\cos(2dx+2c)-59+60\cos(dx+c))\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2048d(-9\cos(2dx+2c)-59+60\cos(dx+c))}$

```
[In] int(1/(5-3*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d*(4*(51/512*tan(1/2*d*x+1/2*c)^3+69/2048*tan(1/2*d*x+1/2*c))/(4*tan(1/2*
d*x+1/2*c)^2+1)^2+59/1024*arctan(2*tan(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

$$\int \frac{1}{(5-3\cos(c+dx))^3}\,dx = -\frac{59\left(9\cos(dx+c)^2-30\cos(dx+c)+25\right)\arctan\left(\frac{5\cos(dx+c)-3}{4\sin(dx+c)}\right)+12\left(45\cos(dx+c)-91\right)\sin(dx+c}{2048\left(9d\cos(dx+c)^2-30d\cos(dx+c)+25d\right)}$$

```
[In] integrate(1/(5-3*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2048*(59*(9*cos(d*x + c)^2 - 30*cos(d*x + c) + 25)*arctan(1/4*(5*cos(d*x
 + c) - 3)/sin(d*x + c)) + 12*(45*cos(d*x + c) - 91)*sin(d*x + c))/(9*d*cos
(d*x + c)^2 - 30*d*cos(d*x + c) + 25*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.43 (sec) , antiderivative size = 364, normalized size of antiderivative = 4.39

$$\int \frac{1}{(5-3\cos(c+dx))^3}\,dx = \begin{cases} \frac{x}{\left(5-3\cosh\left(2\,\mathrm{atanh}\left(\frac{1}{2}\right)\right)\right)^3} \\ \frac{x}{(5-3\cos(c))^3} \\ \frac{944\left(\mathrm{atan}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{16384d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1024d} + \frac{472\left(\mathrm{atan}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{16384d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1024d} + \frac{59\left(\mathrm{atan}\right.}{16384d\tan^4} \end{cases}$$

```
[In] integrate(1/(5-3*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((x/(5 - 3*cosh(2*atanh(1/2)))**3, Eq(c, -d*x - 2*I*atanh(1/2)) |
Eq(c, -d*x + 2*I*atanh(1/2))), (x/(5 - 3*cos(c))**3, Eq(d, 0)), (944*(atan(
2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**
4/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1024*d) + 472
*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d
*x/2)**2/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1024*d
) + 59*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(1638
4*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1024*d) + 408*tan(c/
2 + d*x/2)**3/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1
024*d) + 138*tan(c/2 + d*x/2)/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2
 + d*x/2)**2 + 1024*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{1}{(5-3\cos(c+dx))^3}\,dx = \frac{\frac{6\left(\frac{23\sin(dx+c)}{\cos(dx+c)+1}+\frac{68\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{16\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1}+59\arctan\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}\right)}{1024\,d}$$

```
[In] integrate(1/(5-3*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(6*(23*sin(d*x + c)/(cos(d*x + c) + 1) + 68*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 16*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 1) + 59*arctan(2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{1}{(5-3\cos(c+dx))^3}\,dx = \frac{59\,dx + 59\,c + \frac{12\left(68\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+23\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2} - 118\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{2048\,d}$$

```
[In] integrate(1/(5-3*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2048*(59*d*x + 59*c + 12*(68*tan(1/2*d*x + 1/2*c)^3 + 23*tan(1/2*d*x + 1/
2*c))/(4*tan(1/2*d*x + 1/2*c)^2 + 1)^2 - 118*arctan(sin(d*x + c)/(cos(d*x +
 c) - 3)))/d
```

Mupad [B] (verification not implemented)

Time = 15.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{1}{(5-3\cos(c+dx))^3}\,dx = \frac{59\,\mathrm{atan}\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{1024\,d} - \frac{59\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{1024\,d} + \frac{\frac{51\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2048}+\frac{69\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8192}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\frac{1}{16}\right)}$$

```
[In] int(-1/(3*cos(c + d*x) - 5)^3,x)
```

```
[Out] (59*atan(2*tan(c/2 + (d*x)/2)))/(1024*d) - (59*(atan(tan(c/2 + (d*x)/2)) -
(d*x)/2))/(1024*d) + ((69*tan(c/2 + (d*x)/2))/8192 + (51*tan(c/2 + (d*x)/2)
^3)/2048)/(d*(tan(c/2 + (d*x)/2)^2/2 + tan(c/2 + (d*x)/2)^4 + 1/16))
```

3.25 $\int \frac{1}{(5-3\cos(c+dx))^4}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 108

$$\int \frac{1}{(5-3\cos(c+dx))^4}\,dx = \frac{385x}{32768}+\frac{385\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{16384d}+\frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3}+\frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2}+\frac{311\sin(c+dx)}{8192d(5-3\cos(c+dx))}$$

```
[Out] 385/32768*x+385/16384*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d+1/16*sin(d*x+c)/d
/(5-3*cos(d*x+c))^3+25/512*sin(d*x+c)/d/(5-3*cos(d*x+c))^2+311/8192*sin(d*x
+c)/d/(5-3*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2743, 2833, 12, 2736}

$$\int \frac{1}{(5-3\cos(c+dx))^4}\,dx = \frac{385\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{16384d}+\frac{311\sin(c+dx)}{8192d(5-3\cos(c+dx))}+\frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2}+\frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3}+\frac{385x}{32768}$$

```
[In] Int[(5 - 3*Cos[c + d*x])^(-4),x]
```

```
[Out] (385*x)/32768 + (385*ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])])/(16384*d) + S
in[c + d*x]/(16*d*(5 - 3*Cos[c + d*x])^3) + (25*Sin[c + d*x])/(512*d*(5 - 3
*Cos[c + d*x])^2) + (311*Sin[c + d*x])/(8192*d*(5 - 3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} - \frac{1}{48}\int \frac{-15-6\cos(c+dx)}{(5-3\cos(c+dx))^3}\,dx \\
&= \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} + \frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2} + \frac{\int \frac{186+75\cos(c+dx)}{(5-3\cos(c+dx))^2}\,dx}{1536} \\
&= \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} + \frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2} \\
&\quad + \frac{311\sin(c+dx)}{8192d(5-3\cos(c+dx))} - \frac{\int -\frac{1155}{5-3\cos(c+dx)}\,dx}{24576} \\
&= \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} + \frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2} \\
&\quad + \frac{311\sin(c+dx)}{8192d(5-3\cos(c+dx))} + \frac{385\int \frac{1}{5-3\cos(c+dx)}\,dx}{8192}
\end{aligned}$$

$$= \frac{385x}{32768} + \frac{385\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{16384d} + \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} + \frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2} + \frac{311\sin(c+dx)}{8192d(5-3\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

$$\int \frac{1}{(5-3\cos(c+dx))^4}\,dx = \frac{770\arctan\left(2\tan\left(\frac{1}{2}(c+dx)\right)\right) - \frac{9(4883\sin(c+dx)-2340\sin(2(c+dx))+311\sin(3(c+dx)))}{(-5+3\cos(c+dx))^3}}{32768d}$$

```
[In] Integrate[(5 - 3*Cos[c + d*x])^(-4),x]
```

```
[Out] (770*ArcTan[2*Tan[(c + d*x)/2]] - (9*(4883*Sin[c + d*x] - 2340*Sin[2*(c + d
*x)] + 311*Sin[3*(c + d*x)]))/(-5 + 3*Cos[c + d*x])^3)/(32768*d)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{\frac{\frac{369\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512}+\frac{117\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}+\frac{639\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192}}{\left(4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)^3}+\frac{385\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16384}}{d}$
default	$\frac{\frac{\frac{369\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512}+\frac{117\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}+\frac{639\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192}}{\left(4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)^3}+\frac{385\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16384}}{d}$
risch	$\frac{i\left(10395\,e^{5i(dx+c)}-86625\,e^{4i(dx+c)}+239470\,e^{3i(dx+c)}-218466\,e^{2i(dx+c)}+73575\,e^{i(dx+c)}-8397\right)}{12288d\left(3\,e^{2i(dx+c)}-10\,e^{i(dx+c)}+3\right)^3} - \frac{385i\ln\left(e^{i(dx+c)}-\right.}{32768d}$
parallelrisch	$\frac{385i(770-27\cos(3dx+3c)-981\cos(dx+c)+270\cos(2dx+2c))\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+385i(27\cos(3dx+3c)+981\cos(dx}{32768d(27\cos(3dx+3c)+981\cos(dx}$

```
[In] int(1/(5-3*cos(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(8*(369/4096*tan(1/2*d*x+1/2*c)^5+117/2048*tan(1/2*d*x+1/2*c)^3+639/655
36*tan(1/2*d*x+1/2*c))/(4*tan(1/2*d*x+1/2*c)^2+1)^3+385/16384*arctan(2*tan(
1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{1}{(5-3\cos(c+dx))^4} dx = -\frac{385 \left(27 \cos \left(dx+c\right)^{3}-135 \cos \left(dx+c\right)^{2}+225 \cos \left(dx+c\right)-125\right) \arctan \left(\frac{5 \cos(dx+c)-3}{4 \sin(dx+c)}\right)+36 \left(311 \cos\right.}{32768 \left(27 \, d \cos \left(dx+c\right)^{3}-135 \, d \cos \left(dx+c\right)^{2}+225 \, d \cos \left(dx+\right.\right.}$$

```
[In] integrate(1/(5-3*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/32768*(385*(27*cos(d*x + c)^3 - 135*cos(d*x + c)^2 + 225*cos(d*x + c) -
125)*arctan(1/4*(5*cos(d*x + c) - 3)/sin(d*x + c)) + 36*(311*cos(d*x + c)^2
 - 1170*cos(d*x + c) + 1143)*sin(d*x + c))/(27*d*cos(d*x + c)^3 - 135*d*cos
(d*x + c)^2 + 225*d*cos(d*x + c) - 125*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.85 (sec) , antiderivative size = 597, normalized size of antiderivative = 5.53

$$\int \frac{1}{(5-3\cos(c+dx))^4} dx = \begin{cases} \frac{x}{\left(5-3\cosh\left(2\operatorname{atanh}\left(\frac{1}{2}\right)\right)\right)^4} \\ \frac{x}{\left(5-3\cos\left(c\right)\right)^4} \\ \frac{24640\left(\operatorname{atan}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{1048576d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+786432d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+196608d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+16384d} + \frac{18480\left(\operatorname{atan}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right.}{1048576d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+786432d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+19660} \end{cases}$$

```
[In] integrate(1/(5-3*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((x/(5 - 3*cosh(2*atanh(1/2)))**4, Eq(c, -d*x - 2*I*atanh(1/2)) |
Eq(c, -d*x + 2*I*atanh(1/2))), (x/(5 - 3*cos(c))**4, Eq(d, 0)), (24640*(ata
n(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)
**6/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*
d*tan(c/2 + d*x/2)**2 + 16384*d) + 18480*(atan(2*tan(c/2 + d*x/2)) + pi*flo
or((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(1048576*d*tan(c/2 + d*x/2
)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*
d) + 4620*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*ta
n(c/2 + d*x/2)**2/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2
)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 385*(atan(2*tan(c/2 + d*x/
```

```
2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(1048576*d*tan(c/2 + d*x/2)**6 + 7
86432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 118
08*tan(c/2 + d*x/2)**5/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 +
d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 7488*tan(c/2 + d*x/2)
**3/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*
d*tan(c/2 + d*x/2)**2 + 16384*d) + 1278*tan(c/2 + d*x/2)/(1048576*d*tan(c/2
 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2
+ 16384*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.41

$$\int \frac{1}{(5-3\cos(c+dx))^4}\,dx = \frac{\frac{18\left(\frac{71\sin(dx+c)}{\cos(dx+c)+1}+\frac{416\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{656\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{48\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{64\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1}+385\arctan\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}\right)}{16384\,d}$$

```
[In] integrate(1/(5-3*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/16384*(18*(71*sin(d*x + c)/(cos(d*x + c) + 1) + 416*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 656*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(12*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 48*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 64*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 1) + 385*arctan(2*sin(d*x + c)/(cos(d*x + c
) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \frac{1}{(5-3\cos(c+dx))^4}\,dx = \frac{385\,dx+385\,c+\frac{36\left(656\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+416\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+71\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}-770\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{32768\,d}$$

```
[In] integrate(1/(5-3*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/32768*(385*d*x + 385*c + 36*(656*tan(1/2*d*x + 1/2*c)^5 + 416*tan(1/2*d*x
 + 1/2*c)^3 + 71*tan(1/2*d*x + 1/2*c))/(4*tan(1/2*d*x + 1/2*c)^2 + 1)^3 - 7
70*arctan(sin(d*x + c)/(cos(d*x + c) - 3)))/d
```

Mupad [B] (verification not implemented)

Time = 15.63 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{1}{(5-3\cos(c+dx))^4}\,dx = \frac{385\,\mathrm{atan}\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{16384\,d} - \frac{385\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{16384\,d} + \frac{\frac{369\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{512}+\frac{117\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{256}+\frac{639\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8192}}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^3}$$

```
[In] int(1/(3*cos(c + d*x) - 5)^4,x)
```

```
[Out] (385*atan(2*tan(c/2 + (d*x)/2)))/(16384*d) - (385*(atan(tan(c/2 + (d*x)/2))
 - (d*x)/2))/(16384*d) + ((639*tan(c/2 + (d*x)/2))/8192 + (117*tan(c/2 + (d
*x)/2)^3)/256 + (369*tan(c/2 + (d*x)/2)^5)/512)/(d*(4*tan(c/2 + (d*x)/2)^2
+ 1)^3)
```

3.26 $\int \frac{1}{-5+3\cos(c+dx)}\,dx$

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Mathematica [A] (verified) 210
Maple [A] (verified) 210
Fricas [A] (verification not implemented) 210
Sympy [A] (verification not implemented) 211
Maxima [A] (verification not implemented) 211
Giac [A] (verification not implemented) 211
Mupad [B] (verification not implemented) 212

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{1}{-5+3\cos(c+dx)}\,dx = -\frac{x}{4}-\frac{\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{2d}$$

```
[Out] -1/4*x-1/2*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2737}

$$\int \frac{1}{-5+3\cos(c+dx)}\,dx = -\frac{\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{2d}-\frac{x}{4}$$

```
[In] Int[(-5 + 3*Cos[c + d*x])^(-1),x]
```

```
[Out] -1/4*x - ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])]/(2*d)
```

Rule 2737

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a -
q + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &
& NegQ[a]
```

Rubi steps

$$\text{integral} = -\frac{x}{4}-\frac{\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{2d}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

$$\int \frac{1}{-5+3\cos(c+dx)}\,dx = -\frac{\arctan\left(2\tan\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

```
[In] Integrate[(-5 + 3*Cos[c + d*x])^(-1),x]
[Out] -1/2*ArcTan[2*Tan[(c + d*x)/2]]/d
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.55

method	result	size
derivativedivides	$-\frac{\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}$	18
default	$-\frac{\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}$	18
risch	$-\frac{i\ln\left(\mathrm{e}^{i(dx+c)}-3\right)}{4d}+\frac{i\ln\left(\mathrm{e}^{i(dx+c)}-\frac{1}{3}\right)}{4d}$	38
parallelrisch	$\frac{i\left(\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)-\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)\right)}{4d}$	40

```
[In] int(1/(-5+3*cos(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/2/d*arctan(2*tan(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{-5+3\cos(c+dx)}\,dx = \frac{\arctan\left(\frac{5\,\cos(dx+c)-3}{4\,\sin(dx+c)}\right)}{4\,d}$$

```
[In] integrate(1/(-5+3*cos(d*x+c)),x, algorithm="fricas")
[Out] 1/4*arctan(1/4*(5*cos(d*x + c) - 3)/sin(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{1}{-5+3\cos(c+dx)}\,dx = \begin{cases} -\frac{\operatorname{atan}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor}{2d} & \text{for } d\neq 0 \\ \frac{x}{3\cos(c)-5} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(-5+3*cos(d*x+c)),x)
[Out] Piecewise((-(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/
(2*d), Ne(d, 0)), (x/(3*cos(c) - 5), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{1}{-5+3\cos(c+dx)}\,dx = -\frac{\arctan\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}\right)}{2d}$$

```
[In] integrate(1/(-5+3*cos(d*x+c)),x, algorithm="maxima")
[Out] -1/2*arctan(2*sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{1}{-5+3\cos(c+dx)}\,dx = -\frac{dx+c-2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{4d}$$

```
[In] integrate(1/(-5+3*cos(d*x+c)),x, algorithm="giac")
[Out] -1/4*(d*x + c - 2*arctan(sin(d*x + c)/(cos(d*x + c) - 3)))/d
```

Mupad [B] (verification not implemented)

Time = 14.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{1}{-5+3\cos(c+dx)}\,dx = \frac{\operatorname{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}}{2\,d}-\frac{\operatorname{atan}\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}$$

```
[In] int(1/(3*cos(c + d*x) - 5),x)
[Out] (atan(tan(c/2 + (d*x)/2)) - (d*x)/2)/(2*d) - atan(2*tan(c/2 + (d*x)/2))/(2*
d)
```

3.27 $\int \frac{1}{(-5+3\cos(c+dx))^2}\,dx$

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Mathematica [A] (verified) . . . 214
Maple [A] (verified) . . . 214
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Maxima [A] (verification not implemented) . . . 216
Giac [A] (verification not implemented) . . . 216
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Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{(-5+3\cos(c+dx))^2}\,dx = \frac{5x}{64}+\frac{5\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{32d}+\frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))}$$

```
[Out] 5/64*x+5/32*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d+3/16*sin(d*x+c)/d/(5-3*cos(
d*x+c))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2743, 12, 2737}

$$\int \frac{1}{(-5+3\cos(c+dx))^2}\,dx = \frac{5\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{32d}+\frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))}+\frac{5x}{64}$$

```
[In] Int[(-5 + 3*Cos[c + d*x])^(-2),x]
```

```
[Out] (5*x)/64 + (5*ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])])/(32*d) + (3*Sin[c +
d*x])/(16*d*(5 - 3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2737

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a -
q + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &
& NegQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))} - \frac{1}{16}\int \frac{5}{-5+3\cos(c+dx)}\,dx \\
&= \frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))} - \frac{5}{16}\int \frac{1}{-5+3\cos(c+dx)}\,dx \\
&= \frac{5x}{64} + \frac{5\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{32d} + \frac{3\sin(c+dx)}{16d(5-3\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-5+3\cos(c+dx))^2}\,dx = \frac{5\arctan\left(2\tan\left(\frac{1}{2}(c+dx)\right)\right) - \frac{6\sin(c+dx)}{-5+3\cos(c+dx)}}{32d}$$

```
[In] Integrate[(-5 + 3*Cos[c + d*x])^(-2),x]
[Out] (5*ArcTan[2*Tan[(c + d*x)/2]] - (6*Sin[c + d*x])/(-5 + 3*Cos[c + d*x]))/(32
*d)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{4}\right)}+\frac{5\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}}{d}$	46
default	$\frac{\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{4}\right)}+\frac{5\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}}{d}$	46
parallelrisch	$\frac{(-15i\cos(dx+c)+25i)\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+(15i\cos(dx+c)-25i)\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)-12\sin(dx+c)}{192d\cos(dx+c)-320d}$	82
risch	$\frac{i\left(5\,e^{i(dx+c)}-3\right)}{8d\left(3\,e^{2i(dx+c)}-10\,e^{i(dx+c)}+3\right)}+\frac{5i\ln\left(e^{i(dx+c)}-3\right)}{64d}-\frac{5i\ln\left(e^{i(dx+c)}-\frac{1}{3}\right)}{64d}$	83

```
[In] int(1/(-5+3*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(3/64*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+1/4)+5/32*arctan(2*tan(1
/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{1}{(-5+3\cos(c+dx))^2}\,dx = -\frac{5\,(3\,\cos\left(dx+c\right)-5)\arctan\left(\frac{5\,\cos(dx+c)-3}{4\,\sin(dx+c)}\right)+12\,\sin\left(dx+c\right)}{64\,(3\,d\cos\left(dx+c\right)-5\,d)}$$

```
[In] integrate(1/(-5+3*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/64*(5*(3*cos(d*x + c) - 5)*arctan(1/4*(5*cos(d*x + c) - 3)/sin(d*x + c))
 + 12*sin(d*x + c))/(3*d*cos(d*x + c) - 5*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.31

$$\int \frac{1}{(-5+3\cos(c+dx))^2}\,dx = \begin{cases} \frac{x}{\left(-5+3\cosh\left(2\,\mathrm{atanh}\left(\frac{1}{2}\right)\right)\right)^2} & \text{fo} \\ \frac{x}{(3\cos(c)-5)^2} & \text{fo} \\ \frac{20\left(\mathrm{atan}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{128d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+32d}+\frac{5\left(\mathrm{atan}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{128d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+32d}+\frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+32d} & \text{o} \end{cases}$$

```
[In] integrate(1/(-5+3*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((x/(-5 + 3*cosh(2*atanh(1/2)))**2, Eq(c, -d*x - 2*I*atanh(1/2)) |
 Eq(c, -d*x + 2*I*atanh(1/2))), (x/(3*cos(c) - 5)**2, Eq(d, 0)), (20*(atan(
2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**
2/(128*d*tan(c/2 + d*x/2)**2 + 32*d) + 5*(atan(2*tan(c/2 + d*x/2)) + pi*flo
or((c/2 + d*x/2 - pi/2)/pi))/(128*d*tan(c/2 + d*x/2)**2 + 32*d) + 6*tan(c/2
 + d*x/2)/(128*d*tan(c/2 + d*x/2)**2 + 32*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-5+3\cos(c+dx))^2}\,dx = \frac{\frac{6\,\sin(dx+c)}{\left(\frac{4\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)} + 5\,\arctan\left(\frac{2\,\sin(dx+c)}{\cos(dx+c)+1}\right)}{32\,d}$$

```
[In] integrate(1/(-5+3*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/32*(6*sin(d*x + c)/((4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x
+ c) + 1)) + 5*arctan(2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int \frac{1}{(-5+3\cos(c+dx))^2}\,dx = \frac{5\,dx + 5\,c + \frac{12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} - 10\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{64\,d}$$

```
[In] integrate(1/(-5+3*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(5*d*x + 5*c + 12*tan(1/2*d*x + 1/2*c)/(4*tan(1/2*d*x + 1/2*c)^2 + 1)
- 10*arctan(sin(d*x + c)/(cos(d*x + c) - 3)))/d
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \frac{1}{(-5+3\cos(c+dx))^2}\,dx = \frac{5\,\mathrm{atan}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{32\,d} - \frac{5\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)}{32\,d} + \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{1}{4}\right)}$$

```
[In] int(1/(3*cos(c + d*x) - 5)^2,x)
```

```
[Out] (5*atan(2*tan(c/2 + (d*x)/2)))/(32*d) - (5*(atan(tan(c/2 + (d*x)/2)) - (d*x
)/2))/(32*d) + (3*tan(c/2 + (d*x)/2))/(64*d*(tan(c/2 + (d*x)/2)^2 + 1/4))
```

3.28 $\int \frac{1}{(-5+3\cos(c+dx))^3} dx$

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Mathematica [A] (verified) 220
Maple [A] (verified) 220
Fricas [A] (verification not implemented) 221
Sympy [C] (verification not implemented) 221
Maxima [A] (verification not implemented) 222
Giac [A] (verification not implemented) 222
Mupad [B] (verification not implemented) 222

Optimal result

Integrand size = 12, antiderivative size = 83

$$\int \frac{1}{(-5+3\cos(c+dx))^3} dx = -\frac{59x}{2048} - \frac{59\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{1024d} - \frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2} - \frac{45\sin(c+dx)}{512d(5-3\cos(c+dx))}$$

```
[Out] -59/2048*x-59/1024*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d-3/32*sin(d*x+c)/d/(5
-3*cos(d*x+c))^2-45/512*sin(d*x+c)/d/(5-3*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2743, 2833, 12, 2737}

$$\int \frac{1}{(-5+3\cos(c+dx))^3} dx = -\frac{59\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{1024d} - \frac{45\sin(c+dx)}{512d(5-3\cos(c+dx))} - \frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2} - \frac{59x}{2048}$$

```
[In] Int[(-5 + 3*Cos[c + d*x])^(-3),x]
```

```
[Out] (-59*x)/2048 - (59*ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])])/(1024*d) - (3*S
in[c + d*x])/(32*d*(5 - 3*Cos[c + d*x])^2) - (45*Sin[c + d*x])/(512*d*(5 -
3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2737

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a -
q + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &
& NegQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2} - \frac{1}{32}\int \frac{10+3\cos(c+dx)}{(-5+3\cos(c+dx))^2}\,dx \\
&= -\frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2} - \frac{45\sin(c+dx)}{512d(5-3\cos(c+dx))} + \frac{1}{512}\int \frac{59}{-5+3\cos(c+dx)}\,dx \\
&= -\frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2} - \frac{45\sin(c+dx)}{512d(5-3\cos(c+dx))} + \frac{59}{512}\int \frac{1}{-5+3\cos(c+dx)}\,dx \\
&= -\frac{59x}{2048} - \frac{59\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{1024d} - \frac{3\sin(c+dx)}{32d(5-3\cos(c+dx))^2} - \frac{45\sin(c+dx)}{512d(5-3\cos(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{1}{(-5+3\cos(c+dx))^3}\,dx$$
$$=\frac{-59\arctan\left(2\tan\left(\frac{1}{2}(c+dx)\right)\right)(5-3\cos(c+dx))^2-546\sin(c+dx)+135\sin(2(c+dx))}{1024d(5-3\cos(c+dx))^2}$$

```
[In] Integrate[(-5 + 3*Cos[c + d*x])^(-3),x]
```

```
[Out] (-59*ArcTan[2*Tan[(c + d*x)/2]]*(5 - 3*Cos[c + d*x])^2 - 546*Sin[c + d*x] +
 135*Sin[2*(c + d*x)])/(1024*d*(5 - 3*Cos[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{4\left(\frac{51\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512}+\frac{69\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2048}\right)}{\left(4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)^2}-\frac{59\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}}{d}$
default	$\frac{-\frac{4\left(\frac{51\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512}+\frac{69\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2048}\right)}{\left(4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)^2}-\frac{59\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}}{d}$
risch	$-\frac{3i\left(59\,\mathrm{e}^{3i(dx+c)}-295\,\mathrm{e}^{2i(dx+c)}+241\,\mathrm{e}^{i(dx+c)}-45\right)}{256d\left(3\,\mathrm{e}^{2i(dx+c)}-10\,\mathrm{e}^{i(dx+c)}+3\right)^2}+\frac{59i\ln\left(\mathrm{e}^{i(dx+c)}-\frac{1}{3}\right)}{2048d}-\frac{59i\ln\left(\mathrm{e}^{i(dx+c)}-3\right)}{2048d}$
parallelrisch	$\frac{59i(-9\cos(2dx+2c)-59+60\cos(dx+c))\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+59i(59+9\cos(2dx+2c)-60\cos(dx+c))\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2048d(-9\cos(2dx+2c)-59+60\cos(dx+c))}$

```
[In] int(1/(-5+3*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-4*(51/512*tan(1/2*d*x+1/2*c)^3+69/2048*tan(1/2*d*x+1/2*c))/(4*tan(1/2
*d*x+1/2*c)^2+1)^2-59/1024*arctan(2*tan(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

$$\int \frac{1}{(-5+3\cos(c+dx))^3}\,dx = \frac{59\left(9\,\cos\left(dx+c\right)^2-30\,\cos\left(dx+c\right)+25\right)\arctan\left(\frac{5\,\cos(dx+c)-3}{4\,\sin(dx+c)}\right)+12\left(45\,\cos\left(dx+c\right)-91\right)\sin\left(dx+c\right)}{2048\left(9\,d\cos\left(dx+c\right)^2-30\,d\cos\left(dx+c\right)+25\,d\right)}$$

```
[In] integrate(1/(-5+3*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2048*(59*(9*cos(d*x + c)^2 - 30*cos(d*x + c) + 25)*arctan(1/4*(5*cos(d*x
+ c) - 3)/sin(d*x + c)) + 12*(45*cos(d*x + c) - 91)*sin(d*x + c))/(9*d*cos(
d*x + c)^2 - 30*d*cos(d*x + c) + 25*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 366, normalized size of antiderivative = 4.41

$$\int \frac{1}{(-5+3\cos(c+dx))^3}\,dx = \begin{cases} \frac{x}{\left(-5+3\cosh\left(2\,\mathrm{atanh}\left(\frac{1}{2}\right)\right)\right)^3} \\ \frac{x}{\left(3\cos\left(c\right)-5\right)^3} \\ -\frac{944\left(\mathrm{atan}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{16384d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1024d}-\frac{472\left(\mathrm{atan}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{16384d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1024d}-\frac{59\left(\mathrm{at}\right.}{16384d\,\mathrm{ta}} \end{cases}$$

```
[In] integrate(1/(-5+3*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((x/(-5 + 3*cosh(2*atanh(1/2)))**3, Eq(c, -d*x - 2*I*atanh(1/2)) |
 Eq(c, -d*x + 2*I*atanh(1/2))), (x/(3*cos(c) - 5)**3, Eq(d, 0)), (-944*(ata
n(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)
**4/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1024*d) - 4
72*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 +
 d*x/2)**2/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1024
*d) - 59*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(16
384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 1024*d) - 408*tan(
c/2 + d*x/2)**3/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 +
 1024*d) - 138*tan(c/2 + d*x/2)/(16384*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c
/2 + d*x/2)**2 + 1024*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{1}{(-5+3\cos(c+dx))^3}\,dx = -\frac{\frac{6\left(\frac{23\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{68\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{16\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1}+59\,\arctan\left(\frac{2\,\sin(dx+c)}{\cos(dx+c)+1}\right)}{1024\,d}$$

```
[In] integrate(1/(-5+3*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/1024*(6*(23*sin(d*x + c)/(cos(d*x + c) + 1) + 68*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 16*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 1) + 59*arctan(2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{1}{(-5+3\cos(c+dx))^3}\,dx = -\frac{59\,dx+59\,c+\frac{12\left(68\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+23\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}-118\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{2048\,d}$$

```
[In] integrate(1/(-5+3*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2048*(59*d*x + 59*c + 12*(68*tan(1/2*d*x + 1/2*c)^3 + 23*tan(1/2*d*x + 1
/2*c))/(4*tan(1/2*d*x + 1/2*c)^2 + 1)^2 - 118*arctan(sin(d*x + c)/(cos(d*x
+ c) - 3)))/d
```

Mupad [B] (verification not implemented)

Time = 14.40 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16

$$\int \frac{1}{(-5+3\cos(c+dx))^3}\,dx = \frac{59\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{1024\,d}-\frac{59\,\mathrm{atan}\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{1024\,d}-\frac{\frac{51\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2048}+\frac{69\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8192}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\frac{1}{16}\right)}$$

```
[In] int(1/(3*cos(c + d*x) - 5)^3,x)

[Out] (59*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(1024*d) - (59*atan(2*tan(c/2 + (
d*x)/2)))/(1024*d) - ((69*tan(c/2 + (d*x)/2))/8192 + (51*tan(c/2 + (d*x)/2)
^3)/2048)/(d*(tan(c/2 + (d*x)/2)^2/2 + tan(c/2 + (d*x)/2)^4 + 1/16))
```

3.29 $\int \frac{1}{(-5+3\cos(c+dx))^4}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 108

$$\int \frac{1}{(-5+3\cos(c+dx))^4}\,dx = \frac{385x}{32768} + \frac{385\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{16384d} + \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} + \frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2} + \frac{311\sin(c+dx)}{8192d(5-3\cos(c+dx))}$$

```
[Out] 385/32768*x+385/16384*arctan(sin(d*x+c)/(3-cos(d*x+c)))/d+1/16*sin(d*x+c)/d
/(5-3*cos(d*x+c))^3+25/512*sin(d*x+c)/d/(5-3*cos(d*x+c))^2+311/8192*sin(d*x
+c)/d/(5-3*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2743, 2833, 12, 2737}

$$\int \frac{1}{(-5+3\cos(c+dx))^4}\,dx = \frac{385\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{16384d} + \frac{311\sin(c+dx)}{8192d(5-3\cos(c+dx))} + \frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2} + \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} + \frac{385x}{32768}$$

```
[In] Int[(-5 + 3*Cos[c + d*x])^(-4),x]
```

```
[Out] (385*x)/32768 + (385*ArcTan[Sin[c + d*x]/(3 - Cos[c + d*x])])/(16384*d) + S
in[c + d*x]/(16*d*(5 - 3*Cos[c + d*x])^3) + (25*Sin[c + d*x])/(512*d*(5 - 3
*Cos[c + d*x])^2) + (311*Sin[c + d*x])/(8192*d*(5 - 3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2737

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a -
q + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &
& NegQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} - \frac{1}{48}\int \frac{15+6\cos(c+dx)}{(-5+3\cos(c+dx))^3}\,dx \\
&= \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} + \frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2} + \frac{\int \frac{186+75\cos(c+dx)}{(-5+3\cos(c+dx))^2}\,dx}{1536} \\
&= \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} + \frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2} \\
&\quad + \frac{311\sin(c+dx)}{8192d(5-3\cos(c+dx))} - \frac{\int \frac{1155}{-5+3\cos(c+dx)}\,dx}{24576} \\
&= \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} + \frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2} \\
&\quad + \frac{311\sin(c+dx)}{8192d(5-3\cos(c+dx))} - \frac{385\int \frac{1}{-5+3\cos(c+dx)}\,dx}{8192}
\end{aligned}
$$

$$= \frac{385x}{32768} + \frac{385\arctan\left(\frac{\sin(c+dx)}{3-\cos(c+dx)}\right)}{16384d} + \frac{\sin(c+dx)}{16d(5-3\cos(c+dx))^3} + \frac{25\sin(c+dx)}{512d(5-3\cos(c+dx))^2} + \frac{311\sin(c+dx)}{8192d(5-3\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

$$\int \frac{1}{(-5+3\cos(c+dx))^4}\,dx = \frac{770\arctan\left(2\tan\left(\frac{1}{2}(c+dx)\right)\right) - \frac{9(4883\sin(c+dx)-2340\sin(2(c+dx))+311\sin(3(c+dx)))}{(-5+3\cos(c+dx))^3}}{32768d}$$

```
[In] Integrate[(-5 + 3*Cos[c + d*x])^(-4),x]
[Out] (770*ArcTan[2*Tan[(c + d*x)/2]] - (9*(4883*Sin[c + d*x] - 2340*Sin[2*(c + d
*x)] + 311*Sin[3*(c + d*x)]))/(-5 + 3*Cos[c + d*x])^3)/(32768*d)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{\frac{\frac{369\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512}+\frac{117\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}+\frac{639\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192}}{\left(4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)^3}+\frac{385\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16384}}{d}$
default	$\frac{\frac{\frac{369\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512}+\frac{117\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}+\frac{639\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192}}{\left(4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)^3}+\frac{385\arctan\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16384}}{d}$
risch	$\frac{i\left(10395\,e^{5i(dx+c)}-86625\,e^{4i(dx+c)}+239470\,e^{3i(dx+c)}-218466\,e^{2i(dx+c)}+73575\,e^{i(dx+c)}-8397\right)}{12288d\left(3\,e^{2i(dx+c)}-10\,e^{i(dx+c)}+3\right)^3} - \frac{385i\ln\left(e^{i(dx+c)}-\frac{1}{3}\right)}{32768d}$
parallelrisch	$\frac{385i(770-27\cos(3dx+3c)-981\cos(dx+c)+270\cos(2dx+2c))\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+385i(27\cos(3dx+3c)+981\cos(dx+c}{32768d(27\cos(3dx+3c)+981\cos(dx+c}$

```
[In] int(1/(-5+3*cos(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 1/d*(8*(369/4096*tan(1/2*d*x+1/2*c)^5+117/2048*tan(1/2*d*x+1/2*c)^3+639/655
36*tan(1/2*d*x+1/2*c))/(4*tan(1/2*d*x+1/2*c)^2+1)^3+385/16384*arctan(2*tan(
1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{1}{(-5+3\cos(c+dx))^4}\,dx = -\frac{385\left(27\,\cos\left(dx+c\right)^{3}-135\,\cos\left(dx+c\right)^{2}+225\,\cos\left(dx+c\right)-125\right)\arctan\left(\frac{5\,\cos(dx+c)-3}{4\,\sin(dx+c)}\right)+36\,(311\,\mathrm{co}}{32768\left(27\,d\cos\left(dx+c\right)^{3}-135\,d\cos\left(dx+c\right)^{2}+225\,d\cos\left(dx-\right.\right.}$$

```
[In] integrate(1/(-5+3*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/32768*(385*(27*cos(d*x + c)^3 - 135*cos(d*x + c)^2 + 225*cos(d*x + c) -
125)*arctan(1/4*(5*cos(d*x + c) - 3)/sin(d*x + c)) + 36*(311*cos(d*x + c)^2
 - 1170*cos(d*x + c) + 1143)*sin(d*x + c))/(27*d*cos(d*x + c)^3 - 135*d*cos
(d*x + c)^2 + 225*d*cos(d*x + c) - 125*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.98 (sec) , antiderivative size = 597, normalized size of antiderivative = 5.53

$$\int \frac{1}{(-5+3\cos(c+dx))^4}\,dx = \begin{cases} \frac{x}{\left(-5+3\cosh\left(2\,\mathrm{atanh}\left(\frac{1}{2}\right)\right)\right)^4} \\ \frac{x}{\left(3\cos\left(c\right)-5\right)^4} \\ \frac{24640\left(\mathrm{atan}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{1048576d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+786432d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+196608d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+16384d}+\frac{18480\left(\mathrm{atan}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right.\right.}{1048576d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+786432d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+1966} \end{cases}$$

```
[In] integrate(1/(-5+3*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((x/(-5 + 3*cosh(2*atanh(1/2)))**4, Eq(c, -d*x - 2*I*atanh(1/2)) |
 Eq(c, -d*x + 2*I*atanh(1/2))), (x/(3*cos(c) - 5)**4, Eq(d, 0)), (24640*(at
an(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2
)**6/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608
*d*tan(c/2 + d*x/2)**2 + 16384*d) + 18480*(atan(2*tan(c/2 + d*x/2)) + pi*fl
oor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(1048576*d*tan(c/2 + d*x/
2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384
*d) + 4620*(atan(2*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*t
an(c/2 + d*x/2)**2/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/
2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 385*(atan(2*tan(c/2 + d*x
```

```
/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(1048576*d*tan(c/2 + d*x/2)**6 +
786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 11
808*tan(c/2 + d*x/2)**5/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 +
 d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2 + 16384*d) + 7488*tan(c/2 + d*x/2
)**3/(1048576*d*tan(c/2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608
*d*tan(c/2 + d*x/2)**2 + 16384*d) + 1278*tan(c/2 + d*x/2)/(1048576*d*tan(c/
2 + d*x/2)**6 + 786432*d*tan(c/2 + d*x/2)**4 + 196608*d*tan(c/2 + d*x/2)**2
 + 16384*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.41

$$\int \frac{1}{(-5+3\cos(c+dx))^4}\,dx = \frac{\frac{18\left(\frac{71\sin(dx+c)}{\cos(dx+c)+1}+\frac{416\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{656\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{48\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{64\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1}+385\arctan\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}\right)}{16384\,d}$$

```
[In] integrate(1/(-5+3*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/16384*(18*(71*sin(d*x + c)/(cos(d*x + c) + 1) + 416*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 656*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(12*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 48*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 64*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 1) + 385*arctan(2*sin(d*x + c)/(cos(d*x + c
) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \frac{1}{(-5+3\cos(c+dx))^4}\,dx = \frac{385\,dx+385\,c+\frac{36\left(656\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+416\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+71\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}-770\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)-3}\right)}{32768\,d}$$

```
[In] integrate(1/(-5+3*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/32768*(385*d*x + 385*c + 36*(656*tan(1/2*d*x + 1/2*c)^5 + 416*tan(1/2*d*x
 + 1/2*c)^3 + 71*tan(1/2*d*x + 1/2*c))/(4*tan(1/2*d*x + 1/2*c)^2 + 1)^3 - 7
70*arctan(sin(d*x + c)/(cos(d*x + c) - 3)))/d
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-5+3\cos(c+dx))^4}\,dx = \frac{385\,\mathrm{atan}\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{16384\,d} - \frac{385\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{16384\,d} + \frac{\frac{369\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{512}+\frac{117\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{256}+\frac{639\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8192}}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^3}$$

```
[In] int(1/(3*cos(c + d*x) - 5)^4,x)
```

```
[Out] (385*atan(2*tan(c/2 + (d*x)/2)))/(16384*d) - (385*(atan(tan(c/2 + (d*x)/2))
 - (d*x)/2))/(16384*d) + ((639*tan(c/2 + (d*x)/2))/8192 + (117*tan(c/2 + (d
*x)/2)^3)/256 + (369*tan(c/2 + (d*x)/2)^5)/512)/(d*(4*tan(c/2 + (d*x)/2)^2
+ 1)^3)
```

3.30 $\int \frac{1}{-5-3\cos(c+dx)}\,dx$

Optimal result 230
Rubi [A] (verified) 230
Mathematica [A] (verified) 231
Maple [A] (verified) 231
Fricas [A] (verification not implemented) 231
Sympy [A] (verification not implemented) 232
Maxima [A] (verification not implemented) 232
Giac [A] (verification not implemented) 232
Mupad [B] (verification not implemented) 233

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{-5-3\cos(c+dx)}\,dx = -\frac{x}{4}+\frac{\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{2d}$$

```
[Out] -1/4*x+1/2*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2737}

$$\int \frac{1}{-5-3\cos(c+dx)}\,dx = \frac{\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{2d}-\frac{x}{4}$$

```
[In] Int[(-5 - 3*Cos[c + d*x])^(-1),x]
```

```
[Out] -1/4*x + ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])]/(2*d)
```

Rule 2737

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a -
q + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &
& NegQ[a]
```

Rubi steps

$$\text{integral} = -\frac{x}{4}+\frac{\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{2d}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{1}{-5-3\cos(c+dx)}\,dx = \frac{\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

```
[In] Integrate[(-5 - 3*Cos[c + d*x])^(-1),x]
[Out] ArcTan[2*Cot[(c + d*x)/2]]/(2*d)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$-\frac{\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{2d}$	18
default	$-\frac{\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{2d}$	18
parallelrisch	$\frac{i\left(\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2i\right)\right)}{4d}$	36
risch	$-\frac{i\ln\left(\mathrm{e}^{i(dx+c)}+3\right)}{4d}+\frac{i\ln\left(\mathrm{e}^{i(dx+c)}+\frac{1}{3}\right)}{4d}$	38

```
[In] int(1/(-5-3*cos(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/2/d*arctan(1/2*tan(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{-5-3\cos(c+dx)}\,dx = \frac{\arctan\left(\frac{5\cos(dx+c)+3}{4\sin(dx+c)}\right)}{4\,d}$$

```
[In] integrate(1/(-5-3*cos(d*x+c)),x, algorithm="fricas")
[Out] 1/4*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{1}{-5-3\cos(c+dx)}\,dx = \begin{cases} -\frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor}{2d} & \text{for } d\neq 0 \\ \frac{x}{-3\cos(c)-5} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(-5-3*cos(d*x+c)),x)
[Out] Piecewise((-(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/
(2*d), Ne(d, 0)), (x/(-3*cos(c) - 5), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{-5-3\cos(c+dx)}\,dx = -\frac{\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{2d}$$

```
[In] integrate(1/(-5-3*cos(d*x+c)),x, algorithm="maxima")
[Out] -1/2*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{-5-3\cos(c+dx)}\,dx = -\frac{dx+c-2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{4d}$$

```
[In] integrate(1/(-5-3*cos(d*x+c)),x, algorithm="giac")
[Out] -1/4*(d*x + c - 2*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{1}{-5-3\cos(c+dx)}\,dx = \frac{\operatorname{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}}{2\,d} - \frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{2\,d}$$

```
[In] int(-1/(3*cos(c + d*x) + 5),x)
[Out] (atan(tan(c/2 + (d*x)/2)) - (d*x)/2)/(2*d) - atan(tan(c/2 + (d*x)/2)/2)/(2*
d)
```

3.31 $\int \frac{1}{(-5-3\cos(c+dx))^2}\,dx$

Optimal result . . . 234
Rubi [A] (verified) . . . 234
Mathematica [A] (verified) . . . 235
Maple [A] (verified) . . . 235
Fricas [A] (verification not implemented) . . . 236
Sympy [C] (verification not implemented) . . . 236
Maxima [A] (verification not implemented) . . . 237
Giac [A] (verification not implemented) . . . 237
Mupad [B] (verification not implemented) . . . 238

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(-5-3\cos(c+dx))^2}\,dx=\frac{5x}{64}-\frac{5\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{32d}-\frac{3\sin(c+dx)}{16d(5+3\cos(c+dx))}$$

```
[Out] 5/64*x-5/32*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-3/16*sin(d*x+c)/d/(5+3*cos(
d*x+c))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2743, 12, 2737}

$$\int \frac{1}{(-5-3\cos(c+dx))^2}\,dx=-\frac{5\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{32d}-\frac{3\sin(c+dx)}{16d(3\cos(c+dx)+5)}+\frac{5x}{64}$$

```
[In] Int[(-5 - 3*Cos[c + d*x])^(-2),x]
```

```
[Out] (5*x)/64 - (5*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(32*d) - (3*Sin[c +
d*x])/(16*d*(5 + 3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2737

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a -
q + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &
& NegQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{3\sin(c+dx)}{16d(5+3\cos(c+dx))} - \frac{1}{16}\int \frac{5}{-5-3\cos(c+dx)}\,dx\\ &= -\frac{3\sin(c+dx)}{16d(5+3\cos(c+dx))} - \frac{5}{16}\int \frac{1}{-5-3\cos(c+dx)}\,dx\\ &= \frac{5x}{64} - \frac{5\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{32d} - \frac{3\sin(c+dx)}{16d(5+3\cos(c+dx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{1}{(-5-3\cos(c+dx))^2}\,dx = -\frac{5\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right) + \frac{6\sin(c+dx)}{5+3\cos(c+dx)}}{32d}$$

```
[In] Integrate[(-5 - 3*Cos[c + d*x])^(-2),x]
[Out] -1/32*(5*ArcTan[2*Cot[(c + d*x)/2]] + (6*Sin[c + d*x])/(5 + 3*Cos[c + d*x])
)/d
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)}+\frac{5\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{32}}{d}$	46
default	$\frac{-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)}+\frac{5\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{32}}{d}$	46
parallelrisch	$\frac{(-15i\cos(dx+c)-25i)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+(15i\cos(dx+c)+25i)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2i\right)-12\sin(dx+c)}{192d\cos(dx+c)+320d}$	78
risch	$-\frac{i\left(5\,\mathrm{e}^{i(dx+c)}+3\right)}{8d\left(3\,\mathrm{e}^{2i(dx+c)}+10\,\mathrm{e}^{i(dx+c)}+3\right)}-\frac{5i\ln\left(\mathrm{e}^{i(dx+c)}+\frac{1}{3}\right)}{64d}+\frac{5i\ln\left(\mathrm{e}^{i(dx+c)}+3\right)}{64d}$	83

```
[In] int(1/(-5-3*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(-3/16*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+4)+5/32*arctan(1/2*tan(
1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05

$$\int \frac{1}{(-5-3\cos(c+dx))^2}\,dx = -\frac{5\,(3\,\cos\,(dx+c)+5)\arctan\left(\frac{5\,\cos(dx+c)+3}{4\,\sin(dx+c)}\right)+12\,\sin\,(dx+c)}{64\,(3\,d\cos\,(dx+c)+5\,d)}$$

```
[In] integrate(1/(-5-3*cos(d*x+c))^2,x, algorithm="fricas")
[Out] -1/64*(5*(3*cos(d*x + c) + 5)*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c))
 + 12*sin(d*x + c))/(3*d*cos(d*x + c) + 5*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.39

$$\int \frac{1}{(-5-3\cos(c+dx))^2}\,dx = \begin{cases} \frac{x}{(-5-3\cosh(2\,\mathrm{atanh}\,(2)))^2} & \text{for } c \\ \frac{x}{(-3\cos(c)-5)^2} & \text{for } d \\ \frac{5\left(\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{32d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+128d}+\frac{20\left(\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{32d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+128d}-\frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{32d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+128d} & \text{othe} \end{cases}$$

```
[In] integrate(1/(-5-3*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((x/(-5 - 3*cosh(2*atanh(2)))**2, Eq(c, -d*x - 2*I*atanh(2)) | Eq(
c, -d*x + 2*I*atanh(2))), (x/(-3*cos(c) - 5)**2, Eq(d, 0)), (5*(atan(tan(c/
2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**2/(32*
d*tan(c/2 + d*x/2)**2 + 128*d) + 20*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c
/2 + d*x/2 - pi/2)/pi))/(32*d*tan(c/2 + d*x/2)**2 + 128*d) - 6*tan(c/2 + d*
x/2)/(32*d*tan(c/2 + d*x/2)**2 + 128*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{1}{(-5-3\cos(c+dx))^2}\,dx = -\frac{\frac{6\,\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+4\right)(\cos(dx+c)+1)} - 5\,\arctan\left(\frac{\sin(dx+c)}{2\,(\cos(dx+c)+1)}\right)}{32\,d}$$

```
[In] integrate(1/(-5-3*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/32*(6*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4)*(cos(d*x +
 c) + 1)) - 5*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05

$$\int \frac{1}{(-5-3\cos(c+dx))^2}\,dx = \frac{5\,dx + 5\,c - \frac{12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4} - 10\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{64\,d}$$

```
[In] integrate(1/(-5-3*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(5*d*x + 5*c - 12*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 4) -
10*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

$$\int \frac{1}{(-5-3\cos(c+dx))^2}\,dx = \frac{5\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{32\,d} - \frac{5\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{32\,d} - \frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+4\right)}$$

```
[In] int(1/(3*cos(c + d*x) + 5)^2,x)
```

```
[Out] (5*atan(tan(c/2 + (d*x)/2)/2))/(32*d) - (5*(atan(tan(c/2 + (d*x)/2)) - (d*x
)/2))/(32*d) - (3*tan(c/2 + (d*x)/2))/(16*d*(tan(c/2 + (d*x)/2)^2 + 4))
```

3.32 $\int \frac{1}{(-5-3\cos(c+dx))^3}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 81

$$\int \frac{1}{(-5-3\cos(c+dx))^3}\,dx = -\frac{59x}{2048}+\frac{59\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{1024d} + \frac{3\sin(c+dx)}{32d(5+3\cos(c+dx))^2}+\frac{45\sin(c+dx)}{512d(5+3\cos(c+dx))}$$

```
[Out] -59/2048*x+59/1024*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d+3/32*sin(d*x+c)/d/(5
+3*cos(d*x+c))^2+45/512*sin(d*x+c)/d/(5+3*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2743, 2833, 12, 2737}

$$\int \frac{1}{(-5-3\cos(c+dx))^3}\,dx = \frac{59\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{1024d}+\frac{45\sin(c+dx)}{512d(3\cos(c+dx)+5)} + \frac{3\sin(c+dx)}{32d(3\cos(c+dx)+5)^2}-\frac{59x}{2048}$$

```
[In] Int[(-5 - 3*Cos[c + d*x])^(-3),x]
```

```
[Out] (-59*x)/2048 + (59*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(1024*d) + (3*S
in[c + d*x])/(32*d*(5 + 3*Cos[c + d*x])^2) + (45*Sin[c + d*x])/(512*d*(5 +
3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2737

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a -
q + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &
& NegQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3\sin(c+dx)}{32d(5+3\cos(c+dx))^2} - \frac{1}{32}\int \frac{10-3\cos(c+dx)}{(-5-3\cos(c+dx))^2}\,dx\\ &= \frac{3\sin(c+dx)}{32d(5+3\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(5+3\cos(c+dx))} + \frac{1}{512}\int \frac{59}{-5-3\cos(c+dx)}\,dx\\ &= \frac{3\sin(c+dx)}{32d(5+3\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(5+3\cos(c+dx))} + \frac{59}{512}\int \frac{1}{-5-3\cos(c+dx)}\,dx\\ &= -\frac{59x}{2048} + \frac{59\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{1024d} + \frac{3\sin(c+dx)}{32d(5+3\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(5+3\cos(c+dx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

$$\int \frac{1}{(-5-3\cos(c+dx))^3}\,dx = \frac{59\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)(5+3\cos(c+dx))^2+546\sin(c+dx)+135\sin(2(c+dx))}{1024d(5+3\cos(c+dx))^2}$$

```
[In] Integrate[(-5 - 3*Cos[c + d*x])^(-3),x]
```

```
[Out] (59*ArcTan[2*Cot[(c + d*x)/2]]*(5 + 3*Cos[c + d*x])^2 + 546*Sin[c + d*x] +
135*Sin[2*(c + d*x)])/(1024*d*(5 + 3*Cos[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{-\frac{69\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}}{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^2}-\frac{59\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1024}}{d}$
default	$\frac{-\frac{-\frac{69\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}}{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^2}-\frac{59\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1024}}{d}$
risch	$\frac{3i\left(59\,e^{3i(dx+c)}+295\,e^{2i(dx+c)}+241\,e^{i(dx+c)}+45\right)}{256d\left(3\,e^{2i(dx+c)}+10\,e^{i(dx+c)}+3\right)^2}+\frac{59i\ln\left(e^{i(dx+c)}+\frac{1}{3}\right)}{2048d}-\frac{59i\ln\left(e^{i(dx+c)}+3\right)}{2048d}$
parallelrisch	$\frac{59i(59+9\cos(2dx+2c)+60\cos(dx+c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+59i(-59-9\cos(2dx+2c)-60\cos(dx+c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2048d(59+9\cos(2dx+2c)+60\cos(dx+c))}$

```
[In] int(1/(-5-3*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*(-69/128*tan(1/2*d*x+1/2*c)^3-51/32*tan(1/2*d*x+1/2*c))/(tan(1/2*
d*x+1/2*c)^2+4)^2-59/1024*arctan(1/2*tan(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \frac{1}{(-5-3\cos(c+dx))^3}\,dx = \frac{59\left(9\cos(dx+c)^2+30\cos(dx+c)+25\right)\arctan\left(\frac{5\cos(dx+c)+3}{4\sin(dx+c)}\right)+12\left(45\cos(dx+c)+91\right)\sin(dx+c)}{2048\left(9d\cos(dx+c)^2+30d\cos(dx+c)+25d\right)}$$

```
[In] integrate(1/(-5-3*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2048*(59*(9*cos(d*x + c)^2 + 30*cos(d*x + c) + 25)*arctan(1/4*(5*cos(d*x
+ c) + 3)/sin(d*x + c)) + 12*(45*cos(d*x + c) + 91)*sin(d*x + c))/(9*d*cos(
d*x + c)^2 + 30*d*cos(d*x + c) + 25*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 362, normalized size of antiderivative = 4.47

$$\int \frac{1}{(-5-3\cos(c+dx))^3}\,dx = \begin{cases} \frac{x}{(-5-3\cosh(2\operatorname{atanh}(2)))^3} \\ \frac{x}{(-3\cos(c)-5)^3} \\ -\frac{59\left(\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{1024d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+16384d} - \frac{472\left(\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1024d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+16384d} - \frac{944\left(\operatorname{atan}\right.}{1024d\tan^4\left(\frac{c}{2}\right.} \end{cases}$$

```
[In] integrate(1/(-5-3*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((x/(-5 - 3*cosh(2*atanh(2)))**3, Eq(c, -d*x - 2*I*atanh(2)) | Eq(
c, -d*x + 2*I*atanh(2))), (x/(-3*cos(c) - 5)**3, Eq(d, 0)), (-59*(atan(tan(
c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(1
024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d) - 472*(at
an(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2
)**2/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d) -
944*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(1024*d*
tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384*d) + 138*tan(c/2 +
 d*x/2)**3/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d*x/2)**2 + 16384
*d) + 408*tan(c/2 + d*x/2)/(1024*d*tan(c/2 + d*x/2)**4 + 8192*d*tan(c/2 + d
*x/2)**2 + 16384*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

$$\int \frac{1}{(-5-3\cos(c+dx))^3}\,dx = \frac{\frac{6\left(\frac{68\sin(dx+c)}{\cos(dx+c)+1}+\frac{23\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+16} - 59\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{1024\,d}$$

```
[In] integrate(1/(-5-3*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(6*(68*sin(d*x + c)/(cos(d*x + c) + 1) + 23*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 16) - 59*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{1}{(-5-3\cos(c+dx))^3}\,dx = -\frac{59\,dx+59\,c-\frac{12\left(23\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+68\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\right)^2} - 118\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{2048\,d}$$

```
[In] integrate(1/(-5-3*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2048*(59*d*x + 59*c - 12*(23*tan(1/2*d*x + 1/2*c)^3 + 68*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 4)^2 - 118*arctan(sin(d*x + c)/(cos(d*x +
c) + 3)))/d
```

Mupad [B] (verification not implemented)

Time = 14.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17

$$\int \frac{1}{(-5-3\cos(c+dx))^3}\,dx = \frac{59\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{1024\,d} - \frac{59\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{1024\,d} + \frac{\frac{69\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{512}+\frac{51\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+16\right)}$$

```
[In] int(-1/(3*cos(c + d*x) + 5)^3,x)
```

```
[Out] (59*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(1024*d) - (59*atan(tan(c/2 + (d*
x)/2)/2))/(1024*d) + ((51*tan(c/2 + (d*x)/2))/128 + (69*tan(c/2 + (d*x)/2)^
3)/512)/(d*(8*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 16))
```

3.33 $\int \frac{1}{(-5-3\cos(c+dx))^4}\,dx$

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Mathematica [A] (verified) . . . 247
Maple [A] (verified) . . . 247
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Maxima [A] (verification not implemented) . . . 249
Giac [A] (verification not implemented) . . . 249
Mupad [B] (verification not implemented) . . . 250

Optimal result

Integrand size = 12, antiderivative size = 106

$$\int \frac{1}{(-5-3\cos(c+dx))^4}\,dx = \frac{385x}{32768} - \frac{385\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{16384d} - \frac{\sin(c+dx)}{16d(5+3\cos(c+dx))^3} - \frac{25\sin(c+dx)}{512d(5+3\cos(c+dx))^2} - \frac{311\sin(c+dx)}{8192d(5+3\cos(c+dx))}$$

```
[Out] 385/32768*x-385/16384*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-1/16*sin(d*x+c)/d
/(5+3*cos(d*x+c))^3-25/512*sin(d*x+c)/d/(5+3*cos(d*x+c))^2-311/8192*sin(d*x
+c)/d/(5+3*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2743, 2833, 12, 2737}

$$\int \frac{1}{(-5-3\cos(c+dx))^4}\,dx = -\frac{385\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{16384d} - \frac{311\sin(c+dx)}{8192d(3\cos(c+dx)+5)} - \frac{25\sin(c+dx)}{512d(3\cos(c+dx)+5)^2} - \frac{\sin(c+dx)}{16d(3\cos(c+dx)+5)^3} + \frac{385x}{32768}$$

```
[In] Int[(-5 - 3*Cos[c + d*x])^(-4),x]
```

```
[Out] (385*x)/32768 - (385*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(16384*d) - S
in[c + d*x]/(16*d*(5 + 3*Cos[c + d*x])^3) - (25*Sin[c + d*x])/(512*d*(5 + 3
*Cos[c + d*x])^2) - (311*Sin[c + d*x])/(8192*d*(5 + 3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2737

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a -
q + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &
& NegQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)}{16d(5+3\cos(c+dx))^3}-\frac{1}{48}\int\frac{15-6\cos(c+dx)}{(-5-3\cos(c+dx))^3}\,dx\\
&= -\frac{\sin(c+dx)}{16d(5+3\cos(c+dx))^3}-\frac{25\sin(c+dx)}{512d(5+3\cos(c+dx))^2}+\frac{\int\frac{186-75\cos(c+dx)}{(-5-3\cos(c+dx))^2}\,dx}{1536}\\
&= -\frac{\sin(c+dx)}{16d(5+3\cos(c+dx))^3}-\frac{25\sin(c+dx)}{512d(5+3\cos(c+dx))^2}\\
&\quad-\frac{311\sin(c+dx)}{8192d(5+3\cos(c+dx))}-\frac{\int\frac{1155}{-5-3\cos(c+dx)}\,dx}{24576}\\
&= -\frac{\sin(c+dx)}{16d(5+3\cos(c+dx))^3}-\frac{25\sin(c+dx)}{512d(5+3\cos(c+dx))^2}\\
&\quad-\frac{311\sin(c+dx)}{8192d(5+3\cos(c+dx))}-\frac{385\int\frac{1}{-5-3\cos(c+dx)}\,dx}{8192}
\end{aligned}$$

$$= \frac{385x}{32768} - \frac{385\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{16384d} - \frac{\sin(c+dx)}{16d(5+3\cos(c+dx))^3} - \frac{25\sin(c+dx)}{512d(5+3\cos(c+dx))^2} - \frac{311\sin(c+dx)}{8192d(5+3\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int \frac{1}{(-5-3\cos(c+dx))^4}\,dx = -\frac{770\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right) + \frac{9(4883\sin(c+dx)+2340\sin(2(c+dx))+311\sin(3(c+dx)))}{(5+3\cos(c+dx))^3}}{32768d}$$

```
[In] Integrate[(-5 - 3*Cos[c + d*x])^(-4),x]
```

```
[Out] -1/32768*(770*ArcTan[2*Cot[(c + d*x)/2]] + (9*(4883*Sin[c + d*x] + 2340*Sin
[2*(c + d*x)] + 311*Sin[3*(c + d*x)]))/(5 + 3*Cos[c + d*x])^3)/d
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{\frac{-\frac{639\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}-\frac{117\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{369\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}}{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^3}+\frac{385\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{16384}}{d}$
default	$\frac{\frac{-\frac{639\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}-\frac{117\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{369\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}}{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^3}+\frac{385\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{16384}}{d}$
risch	$-\frac{i\left(10395\,e^{5i(dx+c)}+86625\,e^{4i(dx+c)}+239470\,e^{3i(dx+c)}+218466\,e^{2i(dx+c)}+73575\,e^{i(dx+c)}+8397\right)}{12288d\left(3\,e^{2i(dx+c)}+10\,e^{i(dx+c)}+3\right)^3}+\frac{385i\ln(e^{i(dx+c)}}{32768d}$
parallelrisch	$\frac{385i(-770-27\cos(3dx+3c)-981\cos(dx+c)-270\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+385i(770+27\cos(3dx+3c)+981}{32768d(770+27\cos(3dx+3c)+981\,c}$

```
[In] int(1/(-5-3*cos(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*(-639/1024*tan(1/2*d*x+1/2*c)^5-117/32*tan(1/2*d*x+1/2*c)^3-369/64
*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2+4)^3+385/16384*arctan(1/2*tan(1/
2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14

$$\int \frac{1}{(-5-3\cos(c+dx))^4}\,dx = -\frac{385\left(27\cos(dx+c)^3+135\cos(dx+c)^2+225\cos(dx+c)+125\right)\arctan\left(\frac{5\cos(dx+c)+3}{4\sin(dx+c)}\right)+36\left(311\cos\right.}{32768\left(27\,d\cos(dx+c)^3+135\,d\cos(dx+c)^2+225\,d\cos(dx+\right.}$$

```
[In] integrate(1/(-5-3*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/32768*(385*(27*cos(d*x + c)^3 + 135*cos(d*x + c)^2 + 225*cos(d*x + c) +
125)*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c)) + 36*(311*cos(d*x + c)^2
 + 1170*cos(d*x + c) + 1143)*sin(d*x + c))/(27*d*cos(d*x + c)^3 + 135*d*cos
(d*x + c)^2 + 225*d*cos(d*x + c) + 125*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.15 (sec) , antiderivative size = 595, normalized size of antiderivative = 5.61

$$\int \frac{1}{(-5-3\cos(c+dx))^4}\,dx = \begin{cases} \frac{x}{(-5-3\cosh(2\operatorname{atanh}(2)))^4} \\ \frac{x}{(-3\cos(c)-5)^4} \\ \frac{385\left(\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{16384d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+196608d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+786432d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1048576d}+\frac{4620\left(\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{16384d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+196608d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+786432d} \end{cases}$$

```
[In] integrate(1/(-5-3*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((x/(-5 - 3*cosh(2*atanh(2)))**4, Eq(c, -d*x - 2*I*atanh(2)) | Eq(
c, -d*x + 2*I*atanh(2))), (x/(-3*cos(c) - 5)**4, Eq(d, 0)), (385*(atan(tan(
c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**6/(1
6384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/
2 + d*x/2)**2 + 1048576*d) + 4620*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2
 + d*x/2 - pi/2)/pi))*tan(c/2 + d*x/2)**4/(16384*d*tan(c/2 + d*x/2)**6 + 19
6608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) + 18
480*(atan(tan(c/2 + d*x/2)/2) + pi*floor((c/2 + d*x/2 - pi/2)/pi))*tan(c/2
+ d*x/2)**2/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 7
86432*d*tan(c/2 + d*x/2)**2 + 1048576*d) + 24640*(atan(tan(c/2 + d*x/2)/2)
```

```
+ pi*floor((c/2 + d*x/2 - pi/2)/pi))/(16384*d*tan(c/2 + d*x/2)**6 + 196608*
d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) - 1278*ta
n(c/2 + d*x/2)**5/(16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)*
*4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048576*d) - 7488*tan(c/2 + d*x/2)**3/(
16384*d*tan(c/2 + d*x/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c
/2 + d*x/2)**2 + 1048576*d) - 11808*tan(c/2 + d*x/2)/(16384*d*tan(c/2 + d*x
/2)**6 + 196608*d*tan(c/2 + d*x/2)**4 + 786432*d*tan(c/2 + d*x/2)**2 + 1048
576*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.42

$$\int \frac{1}{(-5-3\cos(c+dx))^4}\,dx = -\frac{\frac{18\left(\frac{656\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{416\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{71\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{48\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{12\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+64}-385\,\arctan\left(\frac{\sin(dx+c)}{2\,(\cos(dx+c)+1)}\right)}{16384\,d}$$

```
[In] integrate(1/(-5-3*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/16384*(18*(656*sin(d*x + c)/(cos(d*x + c) + 1) + 416*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 + 71*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 + 64) - 385*arctan(1/2*sin(d*x + c)/(cos(d*x +
c) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{1}{(-5-3\cos(c+dx))^4}\,dx = \frac{385\,dx+385\,c-\frac{36\left(71\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+416\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+656\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\right)^3}-770\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{32768\,d}$$

```
[In] integrate(1/(-5-3*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/32768*(385*d*x + 385*c - 36*(71*tan(1/2*d*x + 1/2*c)^5 + 416*tan(1/2*d*x
+ 1/2*c)^3 + 656*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 4)^3 - 770
*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{1}{(-5-3\cos(c+dx))^4}\,dx = \frac{385\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{16384\,d} - \frac{385\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{16384\,d} - \frac{\frac{639\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8192}+\frac{117\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{256}+\frac{369\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{512}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\right)^3}$$

```
[In] int(1/(3*cos(c + d*x) + 5)^4,x)
```

```
[Out] (385*atan(tan(c/2 + (d*x)/2)/2))/(16384*d) - (385*(atan(tan(c/2 + (d*x)/2))
 - (d*x)/2))/(16384*d) - ((369*tan(c/2 + (d*x)/2))/512 + (117*tan(c/2 + (d*
x)/2)^3)/256 + (639*tan(c/2 + (d*x)/2)^5)/8192)/(d*(tan(c/2 + (d*x)/2)^2 +
4)^3)
```

3.34 $\int \frac{1}{3+5\cos(c+dx)}\,dx$

Optimal result . . . 251
Rubi [A] (verified) . . . 251
Mathematica [A] (verified) . . . 252
Maple [A] (verified) . . . 252
Fricas [A] (verification not implemented) . . . 253
Sympy [A] (verification not implemented) . . . 253
Maxima [A] (verification not implemented) . . . 254
Giac [A] (verification not implemented) . . . 254
Mupad [B] (verification not implemented) . . . 254

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \frac{1}{3+5\cos(c+dx)}\,dx = -\frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[Out] -1/4*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d+1/4*ln(2*cos(1/2*d*x+1/2
*c)+sin(1/2*d*x+1/2*c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2738, 212}

$$\int \frac{1}{3+5\cos(c+dx)}\,dx = \frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[In] Int[(3 + 5*Cos[c + d*x])^(-1),x]
```

```
[Out] -1/4*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/d + Log[2*Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\text{Subst}\left(\int \frac{1}{8-2x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}\\ &= -\frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}+\frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{1}{3+5\cos(c+dx)}\,dx = -\frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[In] Integrate[(3 + 5*Cos[c + d*x])^(-1),x]
[Out] -1/4*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/d + Log[2*Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]/(4*d)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.51

method	result	size
parallelrisch	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{4d}$	33
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{4}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{4}}{d}$	34
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{4}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{4}}{d}$	34
norman	$-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{4d}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{4d}$	36
risch	$-\frac{\ln\left(e^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5}\right)}{4d}+\frac{\ln\left(e^{i(dx+c)}+\frac{3}{5}+\frac{4i}{5}\right)}{4d}$	40

```
[In] int(1/(3+5*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(ln(tan(1/2*d*x+1/2*c)+2)-ln(tan(1/2*d*x+1/2*c)-2))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{1}{3+5\cos(c+dx)}\,dx = \frac{\log\left(\frac{3}{2}\cos\left(dx+c\right)+2\sin\left(dx+c\right)+\frac{5}{2}\right)-\log\left(\frac{3}{2}\cos\left(dx+c\right)-2\sin\left(dx+c\right)+\frac{5}{2}\right)}{8\,d}$$

```
[In] integrate(1/(3+5*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(log(3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2) - log(3/2*cos(d*x + c) -
2*sin(d*x + c) + 5/2))/d
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63

$$\int \frac{1}{3+5\cos(c+dx)}\,dx = \begin{cases} -\frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{4d}+\frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)}{4d} & \text{for}\ d\neq 0\\ \frac{x}{5\cos\left(c\right)+3} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(3+5*cos(d*x+c)),x)
```

```
[Out] Piecewise((-log(tan(c/2 + d*x/2) - 2)/(4*d) + log(tan(c/2 + d*x/2) + 2)/(4*
d), Ne(d, 0)), (x/(5*cos(c) + 3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{1}{3+5\cos(c+dx)}\,dx = \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right)-\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{4\,d}$$

```
[In] integrate(1/(3+5*cos(d*x+c)),x, algorithm="maxima")
[Out] 1/4*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{1}{3+5\cos(c+dx)}\,dx = \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)-\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{4\,d}$$

```
[In] integrate(1/(3+5*cos(d*x+c)),x, algorithm="giac")
[Out] 1/4*(log(abs(tan(1/2*d*x + 1/2*c) + 2)) - log(abs(tan(1/2*d*x + 1/2*c) - 2)
))/d
```

Mupad [B] (verification not implemented)

Time = 14.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.26

$$\int \frac{1}{3+5\cos(c+dx)}\,dx = \frac{\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{2\,d}$$

```
[In] int(1/(5*cos(c + d*x) + 3),x)
[Out] atanh(tan(c/2 + (d*x)/2)/2)/(2*d)
```

3.35 $\int \frac{1}{(3+5\cos(c+dx))^2}\,dx$

Optimal result . . . 255
Rubi [A] (verified) . . . 255
Mathematica [A] (verified) . . . 257
Maple [A] (verified) . . . 257
Fricas [A] (verification not implemented) . . . 258
Sympy [B] (verification not implemented) . . . 258
Maxima [A] (verification not implemented) . . . 259
Giac [A] (verification not implemented) . . . 259
Mupad [B] (verification not implemented) . . . 259

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{1}{(3+5\cos(c+dx))^2}\,dx = \frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{5\sin(c+dx)}{16d(3+5\cos(c+dx))}$$

```
[Out] 3/64*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-3/64*ln(2*cos(1/2*d*x+1/
2*c)+sin(1/2*d*x+1/2*c))/d+5/16*sin(d*x+c)/d/(3+5*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2743, 12, 2738, 212}

$$\int \frac{1}{(3+5\cos(c+dx))^2}\,dx = \frac{5\sin(c+dx)}{16d(5\cos(c+dx)+3)} + \frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{64d}$$

```
[In] Int[(3 + 5*Cos[c + d*x])^(-2),x]
```

```
[Out] (3*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(64*d) - (3*Log[2*Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]])/(64*d) + (5*Sin[c + d*x])/(16*d*(3 + 5*Cos[c +
d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{5\sin(c+dx)}{16d(3+5\cos(c+dx))}+\frac{1}{16}\int -\frac{3}{3+5\cos(c+dx)}\,dx\\
&= \frac{5\sin(c+dx)}{16d(3+5\cos(c+dx))}-\frac{3}{16}\int \frac{1}{3+5\cos(c+dx)}\,dx\\
&= \frac{5\sin(c+dx)}{16d(3+5\cos(c+dx))}-\frac{3\text{Subst}\left(\int \frac{1}{8-2x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{8d}\\
&= \frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d}\\
&\quad -\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d}+\frac{5\sin(c+dx)}{16d(3+5\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.59

$$\int \frac{1}{(3+5\cos(c+dx))^2}\,dx = \frac{9\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+15\cos(c+dx)\left(\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-}{64d(3+5\cos}$$

```
[In] Integrate[(3 + 5*Cos[c + d*x])^(-2),x]
```

```
[Out] (9*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 15*Cos[c + d*x]*(Log[2*Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]) - 9*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 20*Sin[c + d*x])/(64*d
*(3 + 5*Cos[c + d*x]))
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{-\frac{5}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{64}-\frac{5}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{64}}{d}$
default	$\frac{-\frac{5}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{64}-\frac{5}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{64}}{d}$
norman	$-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-4\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{64d}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{64d}$
risch	$\frac{i\left(3\,e^{i(dx+c)}+5\right)}{8d\left(5\,e^{2i(dx+c)}+6\,e^{i(dx+c)}+5\right)}-\frac{3\ln\left(e^{i(dx+c)}+\frac{3}{5}+\frac{4i}{5}\right)}{64d}+\frac{3\ln\left(e^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5}\right)}{64d}$
parallelrisch	$\frac{15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)\cos(dx+c)-15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)\cos(dx+c)+20\sin(dx+c)+9\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)-9\ln\left(\tan\right.}{64d(3+5\cos(dx+c))}$

```
[In] int(1/(3+5*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-5/32/(tan(1/2*d*x+1/2*c)+2)-3/64*ln(tan(1/2*d*x+1/2*c)+2)-5/32/(tan(1
/2*d*x+1/2*c)-2)+3/64*ln(tan(1/2*d*x+1/2*c)-2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{1}{(3+5\cos(c+dx))^2}\,dx = -\frac{3\,(5\,\cos\left(dx+c\right)+3)\log\left(\frac{3}{2}\,\cos\left(dx+c\right)+2\,\sin\left(dx+c\right)+\frac{5}{2}\right)-3\,(5\,\cos\left(dx+c\right)+3)\log\left(\frac{3}{2}\,\cos\left(dx+c\right.\right.}{128\,(5\,d\cos\left(dx+c\right)+3\,d)}$$

```
[In] integrate(1/(3+5*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/128*(3*(5*cos(d*x + c) + 3)*log(3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2)
 - 3*(5*cos(d*x + c) + 3)*log(3/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) - 40
*sin(d*x + c))/(5*d*cos(d*x + c) + 3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(78) = 156.

Time = 0.66 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.53

$$\int \frac{1}{(3+5\cos(c+dx))^2}\,dx = \begin{cases} \frac{x}{(5\cos\left(2\operatorname{atan}\left(2\right)\right)+3)^2} \\ \frac{x}{(5\cos\left(c\right)+3)^2} \\ \frac{3\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} - \frac{12\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{64d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} - \frac{3\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} + \frac{12\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)}{64d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} - \end{cases}$$

```
[In] integrate(1/(3+5*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((x/(5*cos(2*atan(2)) + 3)**2, Eq(c, -d*x - 2*atan(2)) | Eq(c, -d*
x + 2*atan(2))), (x/(5*cos(c) + 3)**2, Eq(d, 0)), (3*log(tan(c/2 + d*x/2) -
 2)*tan(c/2 + d*x/2)**2/(64*d*tan(c/2 + d*x/2)**2 - 256*d) - 12*log(tan(c/2
 + d*x/2) - 2)/(64*d*tan(c/2 + d*x/2)**2 - 256*d) - 3*log(tan(c/2 + d*x/2)
+ 2)*tan(c/2 + d*x/2)**2/(64*d*tan(c/2 + d*x/2)**2 - 256*d) + 12*log(tan(c/
2 + d*x/2) + 2)/(64*d*tan(c/2 + d*x/2)**2 - 256*d) - 20*tan(c/2 + d*x/2)/(6
4*d*tan(c/2 + d*x/2)**2 - 256*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int \frac{1}{(3+5\cos(c+dx))^2}\,dx = -\frac{\frac{20\,\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-4\right)(\cos(dx+c)+1)} + 3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right) - 3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{64\,d}$$

```
[In] integrate(1/(3+5*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/64*(20*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4)*(cos(d*x
+ c) + 1)) + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 3*log(sin(d*x + c
)/(cos(d*x + c) + 1) - 2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.69

$$\int \frac{1}{(3+5\cos(c+dx))^2}\,dx = -\frac{\frac{20\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4} + 3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right) - 3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{64\,d}$$

```
[In] integrate(1/(3+5*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/64*(20*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 4) + 3*log(abs(tan
(1/2*d*x + 1/2*c) + 2)) - 3*log(abs(tan(1/2*d*x + 1/2*c) - 2)))/d
```

Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int \frac{1}{(3+5\cos(c+dx))^2}\,dx = -\frac{3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{32\,d} - \frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\right)}$$

```
[In] int(1/(5*cos(c + d*x) + 3)^2,x)
```

```
[Out] - (3*atanh(tan(c/2 + (d*x)/2)/2))/(32*d) - (5*tan(c/2 + (d*x)/2))/(16*d*(ta
n(c/2 + (d*x)/2)^2 - 4))
```

3.36 $\int \frac{1}{(3+5\cos(c+dx))^3} dx$

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Optimal result

Integrand size = 12, antiderivative size = 115

$$\int \frac{1}{(3+5\cos(c+dx))^3} dx = -\frac{43\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{43\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{5\sin(c+dx)}{32d(3+5\cos(c+dx))^2} - \frac{45\sin(c+dx)}{512d(3+5\cos(c+dx))}$$

```
[Out] -43/2048*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d+43/2048*ln(2*cos(1/2
*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+5/32*sin(d*x+c)/d/(3+5*cos(d*x+c))^2-45/5
12*sin(d*x+c)/d/(3+5*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2743, 2833, 12, 2738, 212}

$$\int \frac{1}{(3+5\cos(c+dx))^3} dx = -\frac{45\sin(c+dx)}{512d(5\cos(c+dx)+3)} + \frac{5\sin(c+dx)}{32d(5\cos(c+dx)+3)^2} - \frac{43\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{43\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}$$

```
[In] Int[(3 + 5*Cos[c + d*x])^(-3),x]
```

```
[Out] (-43*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2048*d) + (43*Log[2*Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]])/(2048*d) + (5*Sin[c + d*x])/(32*d*(3 + 5*C
os[c + d*x])^2) - (45*Sin[c + d*x])/(512*d*(3 + 5*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{5\sin(c+dx)}{32d(3+5\cos(c+dx))^2}+\frac{1}{32}\int\frac{-6+5\cos(c+dx)}{(3+5\cos(c+dx))^2}\,dx\\
&= \frac{5\sin(c+dx)}{32d(3+5\cos(c+dx))^2}-\frac{45\sin(c+dx)}{512d(3+5\cos(c+dx))}+\frac{1}{512}\int\frac{43}{3+5\cos(c+dx)}\,dx
\end{aligned}
$$

$$= \frac{5\sin(c+dx)}{32d(3+5\cos(c+dx))^2} - \frac{45\sin(c+dx)}{512d(3+5\cos(c+dx))} + \frac{43}{512}\int \frac{1}{3+5\cos(c+dx)}\,dx$$

$$= \frac{5\sin(c+dx)}{32d(3+5\cos(c+dx))^2} - \frac{45\sin(c+dx)}{512d(3+5\cos(c+dx))} + \frac{43\text{Subst}\left(\int \frac{1}{8-2x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{256d}$$

$$= -\frac{43\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{43\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{5\sin(c+dx)}{32d(3+5\cos(c+dx))^2} - \frac{45\sin(c+dx)}{512d(3+5\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.89

$$\int \frac{1}{(3+5\cos(c+dx))^3}\,dx = -\frac{43\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{43\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{5}{512d\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{45\sin\left(\frac{1}{2}(c+dx)\right)}{2048d\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{5}{512d\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{45\sin\left(\frac{1}{2}(c+dx)\right)}{2048d\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

```
[In] Integrate[(3 + 5*Cos[c + d*x])^(-3),x]
```

```
[Out] (-43*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2048*d) + (43*Log[2*Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]])/(2048*d) + 5/(512*d*(2*Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])^2) - (45*Sin[(c + d*x)/2])/(2048*d*(2*Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])) - 5/(512*d*(2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) -
(45*Sin[(c + d*x)/2])/(2048*d*(2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

method	result
norman	$\frac{-\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{128d}+\frac{85\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-4\right)^2}-\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{2048d}+\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{2048d}$
derivativedivides	$\frac{\frac{25}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}+\frac{85}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{2048}-\frac{25}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}+\frac{85}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+\frac{43\ln}{}}{d}$
default	$\frac{\frac{25}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}+\frac{85}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{2048}-\frac{25}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}+\frac{85}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+\frac{43\ln}{}}{d}$
risch	$-\frac{i\left(215\,e^{3i(dx+c)}+387\,e^{2i(dx+c)}+325\,e^{i(dx+c)}+225\right)}{256d\left(5\,e^{2i(dx+c)}+6\,e^{i(dx+c)}+5\right)^2}-\frac{43\ln\left(e^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5}\right)}{2048d}+\frac{43\ln\left(e^{i(dx+c)}+\frac{3}{5}+\frac{4i}{5}\right)}{2048d}$
parallelrisch	$\frac{(-2580\cos(dx+c)-1075\cos(2dx+2c)-1849)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)+(2580\cos(dx+c)+1075\cos(2dx+2c)+1849)\ln\left(\tan\right.}{2048d(43+25\cos(2dx+2c)+60\cos(dx+c))}$

```
[In] int(1/(3+5*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-35/128/d*tan(1/2*d*x+1/2*c)+85/512/d*tan(1/2*d*x+1/2*c)^3)/(tan(1/2*d*x+1
/2*c)^2-4)^2-43/2048/d*ln(tan(1/2*d*x+1/2*c)-2)+43/2048/d*ln(tan(1/2*d*x+1/
2*c)+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.12

$$\int \frac{1}{(3+5\cos(c+dx))^3}\,dx$$
$$=\frac{43\left(25\cos\left(dx+c\right)^2+30\cos\left(dx+c\right)+9\right)\log\left(\frac{3}{2}\cos\left(dx+c\right)+2\sin\left(dx+c\right)+\frac{5}{2}\right)-43\left(25\cos\left(dx+c\right.\right.}{4096\left(25\,d\cos\left(dx+c\right)^2\right.}$$

```
[In] integrate(1/(3+5*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4096*(43*(25*cos(d*x + c)^2 + 30*cos(d*x + c) + 9)*log(3/2*cos(d*x + c) +
 2*sin(d*x + c) + 5/2) - 43*(25*cos(d*x + c)^2 + 30*cos(d*x + c) + 9)*log(3
/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) - 40*(45*cos(d*x + c) + 11)*sin(d*x
 + c))/(25*d*cos(d*x + c)^2 + 30*d*cos(d*x + c) + 9*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(102) = 204$.

Time = 1.18 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.12

$$\int \frac{1}{(3+5\cos(c+dx))^3}\,dx = \begin{cases} \frac{x}{(5\cos(2\operatorname{atan}(2))+3)^3} \\ \frac{x}{(5\cos(c)+3)^3} \\ -\frac{43\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2048d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+32768d} + \frac{344\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2048d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+32768d} - \frac{688\log\left(\tan\right.}{2048d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-163} \end{cases}$$

```
[In] integrate(1/(3+5*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((x/(5*cos(2*atan(2)) + 3)**3, Eq(c, -d*x - 2*atan(2)) | Eq(c, -d*
x + 2*atan(2))), (x/(5*cos(c) + 3)**3, Eq(d, 0)), (-43*log(tan(c/2 + d*x/2)
 - 2)*tan(c/2 + d*x/2)**4/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d
*x/2)**2 + 32768*d) + 344*log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**2/(20
48*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) - 688*log
(tan(c/2 + d*x/2) - 2)/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/
2)**2 + 32768*d) + 43*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**4/(2048*d
*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) - 344*log(tan
(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**2/(2048*d*tan(c/2 + d*x/2)**4 - 16384*
d*tan(c/2 + d*x/2)**2 + 32768*d) + 688*log(tan(c/2 + d*x/2) + 2)/(2048*d*ta
n(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) + 340*tan(c/2 +
d*x/2)**3/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768
*d) - 560*tan(c/2 + d*x/2)/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 +
d*x/2)**2 + 32768*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.17

$$\int \frac{1}{(3+5\cos(c+dx))^3}\,dx = \frac{\frac{20\left(\frac{28\sin(dx+c)}{\cos(dx+c)+1}-\frac{17\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-16} + 43\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right) - 43\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{2048\,d}$$

```
[In] integrate(1/(3+5*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(20*(28*sin(d*x + c)/(cos(d*x + c) + 1) - 17*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 16) + 43*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 43*lo
g(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\int \frac{1}{(3+5\cos(c+dx))^3}\,dx = \frac{\frac{20\left(17\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-28\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\right)^2}+43\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)-43\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{2048\,d}$$

```
[In] integrate(1/(3+5*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2048*(20*(17*tan(1/2*d*x + 1/2*c)^3 - 28*tan(1/2*d*x + 1/2*c))/(tan(1/2*d
*x + 1/2*c)^2 - 4)^2 + 43*log(abs(tan(1/2*d*x + 1/2*c) + 2)) - 43*log(abs(t
an(1/2*d*x + 1/2*c) - 2)))/d
```

Mupad [B] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\int \frac{1}{(3+5\cos(c+dx))^3}\,dx = \frac{43\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{1024\,d} - \frac{\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128}-\frac{85\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{512}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+16\right)}$$

```
[In] int(1/(5*cos(c + d*x) + 3)^3,x)
```

```
[Out] (43*atanh(tan(c/2 + (d*x)/2)/2))/(1024*d) - ((35*tan(c/2 + (d*x)/2))/128 -
(85*tan(c/2 + (d*x)/2)^3)/512)/(d*(tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x)
/2)^2 + 16))
```

3.37 $\int \frac{1}{(3+5\cos(c+dx))^4} \, dx$

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Optimal result

Integrand size = 12, antiderivative size = 140

$$\begin{aligned}\int \frac{1}{(3+5\cos(c+dx))^4} \, dx = & \frac{279 \log \left(2 \cos \left(\frac{1}{2}(c+dx)\right)-\sin \left(\frac{1}{2}(c+dx)\right)\right)}{32768 d} \\ & -\frac{279 \log \left(2 \cos \left(\frac{1}{2}(c+dx)\right)+\sin \left(\frac{1}{2}(c+dx)\right)\right)}{32768 d} \\ & +\frac{5 \sin(c+dx)}{48 d (3+5\cos(c+dx))^3}-\frac{25 \sin(c+dx)}{512 d (3+5\cos(c+dx))^2} \\ & +\frac{995 \sin(c+dx)}{24576 d (3+5\cos(c+dx))}\end{aligned}$$

```
[Out] 279/32768*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-279/32768*ln(2*cos(
1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+5/48*sin(d*x+c)/d/(3+5*cos(d*x+c))^3-2
5/512*sin(d*x+c)/d/(3+5*cos(d*x+c))^2+995/24576*sin(d*x+c)/d/(3+5*cos(d*x+c
))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used

$= \{2743, 2833, 12, 2738, 212\}$

$$\int \frac{1}{(3+5\cos(c+dx))^4}\,dx = \frac{995\sin(c+dx)}{24576d(5\cos(c+dx)+3)} - \frac{25\sin(c+dx)}{512d(5\cos(c+dx)+3)^2} + \frac{5\sin(c+dx)}{48d(5\cos(c+dx)+3)^3} + \frac{279\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} - \frac{279\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}$$

```
[In] Int[(3 + 5*Cos[c + d*x])^(-4),x]
```

```
[Out] (279*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(32768*d) - (279*Log[2*Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]])/(32768*d) + (5*Sin[c + d*x])/(48*d*(3 +
5*Cos[c + d*x])^3) - (25*Sin[c + d*x])/(512*d*(3 + 5*Cos[c + d*x])^2) + (99
5*Sin[c + d*x])/(24576*d*(3 + 5*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
```

```
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{5\sin(c+dx)}{48d(3+5\cos(c+dx))^3}+\frac{1}{48}\int\frac{-9+10\cos(c+dx)}{(3+5\cos(c+dx))^3}\,dx\\
&= \frac{5\sin(c+dx)}{48d(3+5\cos(c+dx))^3}-\frac{25\sin(c+dx)}{512d(3+5\cos(c+dx))^2}+\frac{\int\frac{154-75\cos(c+dx)}{(3+5\cos(c+dx))^2}\,dx}{1536}\\
&= \frac{5\sin(c+dx)}{48d(3+5\cos(c+dx))^3}-\frac{25\sin(c+dx)}{512d(3+5\cos(c+dx))^2}\\
&\quad+\frac{995\sin(c+dx)}{24576d(3+5\cos(c+dx))}+\frac{\int-\frac{837}{3+5\cos(c+dx)}\,dx}{24576}\\
&= \frac{5\sin(c+dx)}{48d(3+5\cos(c+dx))^3}-\frac{25\sin(c+dx)}{512d(3+5\cos(c+dx))^2}\\
&\quad+\frac{995\sin(c+dx)}{24576d(3+5\cos(c+dx))}-\frac{279\int\frac{1}{3+5\cos(c+dx)}\,dx}{8192}\\
&= \frac{5\sin(c+dx)}{48d(3+5\cos(c+dx))^3}-\frac{25\sin(c+dx)}{512d(3+5\cos(c+dx))^2}\\
&\quad+\frac{995\sin(c+dx)}{24576d(3+5\cos(c+dx))}-\frac{279\text{Subst}\left(\int\frac{1}{8-2x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{4096d}\\
&= \frac{279\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}\\
&\quad-\frac{279\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}+\frac{5\sin(c+dx)}{48d(3+5\cos(c+dx))^3}\\
&\quad-\frac{25\sin(c+dx)}{512d(3+5\cos(c+dx))^2}+\frac{995\sin(c+dx)}{24576d(3+5\cos(c+dx))}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 296 vs. $2(140) = 280$.

Time = 0.31 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.11

$$\int \frac{1}{(3+5\cos(c+dx))^4}\,dx$$
$$= \frac{467046\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+104625\cos(3(c+dx))\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(\right.\right.}{}$$

```
[In] Integrate[(3 + 5*Cos[c + d*x])^(-4),x]
```

```
[Out] (467046*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 104625*Cos[3*(c + d*x)
]*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 765855*Cos[c + d*x]*(Log[2*C
os[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]]) + 376650*Cos[2*(c + d*x)]*(Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 467046*Log[2*Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] - 104625*Cos[3*(c + d*x)]*Log[2*Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] + 226140*Sin[c + d*x] + 190800*Sin[2*(c + d*x)] + 99500*
Sin[3*(c + d*x)])/(393216*d*(3 + 5*Cos[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.71

method	result
norman	$\frac{-\frac{295\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512d}+\frac{265\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768d}-\frac{745\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8192d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-4\right)^3}+\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{32768d}-\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{32768d}$
derivativedivides	$\frac{-\frac{125}{6144\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^3}-\frac{175}{4096\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}-\frac{745}{16384\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{32768}-\frac{125}{6144\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^3}}{d}$
default	$\frac{-\frac{125}{6144\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^3}-\frac{175}{4096\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}-\frac{745}{16384\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{32768}-\frac{125}{6144\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^3}}{d}$
risch	$\frac{i\left(20925\,e^{5i(dx+c)}+62775\,e^{4i(dx+c)}+111042\,e^{3i(dx+c)}+119310\,e^{2i(dx+c)}+68625\,e^{i(dx+c)}+24875\right)}{12288d\left(5\,e^{2i(dx+c)}+6\,e^{i(dx+c)}+5\right)^3}+\frac{279\ln(e^{i(dx+c)}+}{32768d}$
parallelrisch	$\frac{(765855\cos(dx+c)+376650\cos(2dx+2c)+104625\cos(3dx+3c)+467046)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)+(-765855\cos(dx+c)-3}{98304d(558+125\cos(3dx-}$

```
[In] int(1/(3+5*cos(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-295/512/d*tan(1/2*d*x+1/2*c)+265/768/d*tan(1/2*d*x+1/2*c)^3-745/8192/d*ta
n(1/2*d*x+1/2*c)^5)/(tan(1/2*d*x+1/2*c)^2-4)^3+279/32768/d*ln(tan(1/2*d*x+1
/2*c)-2)-279/32768/d*ln(tan(1/2*d*x+1/2*c)+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.21

$$\int \frac{1}{(3+5\cos(c+dx))^4}\,dx = -\frac{837\left(125\cos(dx+c)^3+225\cos(dx+c)^2+135\cos(dx+c)+27\right)\log\left(\frac{3}{2}\cos(dx+c)+2\sin(dx+c)\right.}{}$$

```
[In] integrate(1/(3+5*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/196608*(837*(125*cos(d*x + c)^3 + 225*cos(d*x + c)^2 + 135*cos(d*x + c)
+ 27)*log(3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2) - 837*(125*cos(d*x + c)^
3 + 225*cos(d*x + c)^2 + 135*cos(d*x + c) + 27)*log(3/2*cos(d*x + c) - 2*si
n(d*x + c) + 5/2) - 40*(4975*cos(d*x + c)^2 + 4770*cos(d*x + c) + 1583)*sin
(d*x + c))/(125*d*cos(d*x + c)^3 + 225*d*cos(d*x + c)^2 + 135*d*cos(d*x + c
) + 27*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. 2(126) = 252.

Time = 2.46 (sec) , antiderivative size = 813, normalized size of antiderivative = 5.81

$$\int \frac{1}{(3+5\cos(c+dx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(3+5*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((x/(5*cos(2*atan(2)) + 3)**4, Eq(c, -d*x - 2*atan(2)) | Eq(c, -d*
x + 2*atan(2))), (x/(5*cos(c) + 3)**4, Eq(d, 0)), (837*log(tan(c/2 + d*x/2)
 - 2)*tan(c/2 + d*x/2)**6/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2
+ d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 10044*log(tan(c/
2 + d*x/2) - 2)*tan(c/2 + d*x/2)**4/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*
d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) + 40176*
log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**2/(98304*d*tan(c/2 + d*x/2)**6
- 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d
) - 53568*log(tan(c/2 + d*x/2) - 2)/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*
d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 837*lo
g(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**6/(98304*d*tan(c/2 + d*x/2)**6 -
1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d)
+ 10044*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**4/(98304*d*tan(c/2 + d*
x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6
291456*d) - 40176*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**2/(98304*d*ta
```

```
n(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x
/2)**2 - 6291456*d) + 53568*log(tan(c/2 + d*x/2) + 2)/(98304*d*tan(c/2 + d*
x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6
291456*d) - 8940*tan(c/2 + d*x/2)**5/(98304*d*tan(c/2 + d*x/2)**6 - 1179648
*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) + 33920
*tan(c/2 + d*x/2)**3/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x
/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 56640*tan(c/2 + d*x/2
)/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*
tan(c/2 + d*x/2)**2 - 6291456*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.24

$$\int \frac{1}{(3+5\cos(c+dx))^4}\,dx = -\frac{\frac{20\left(\frac{2832\sin(dx+c)}{\cos(dx+c)+1} - \frac{1696\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{447\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{12\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - 64} + 837\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right) - 837\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{98304\,d}$$

```
[In] integrate(1/(3+5*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/98304*(20*(2832*sin(d*x + c)/(cos(d*x + c) + 1) - 1696*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 + 447*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 - 64) + 837*log(sin(d*x + c)/(cos(d*x + c) +
 1) + 2) - 837*log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

$$\int \frac{1}{(3+5\cos(c+dx))^4}\,dx = -\frac{\frac{20\left(447\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 1696\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 2832\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4\right)^3} + 837\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right|\right) - 837\log(|\tan}{98304\,d}$$

```
[In] integrate(1/(3+5*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/98304*(20*(447*tan(1/2*d*x + 1/2*c)^5 - 1696*tan(1/2*d*x + 1/2*c)^3 + 28
32*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 4)^3 + 837*log(abs(tan(1
/2*d*x + 1/2*c) + 2)) - 837*log(abs(tan(1/2*d*x + 1/2*c) - 2)))/d
```

Mupad [B] (verification not implemented)

Time = 16.71 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.73

$$\int \frac{1}{(3+5\cos(c+dx))^4}\,dx = -\frac{279\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{16384\,d} - \frac{\frac{745\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8192}-\frac{265\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{768}+\frac{295\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{512}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-64\right)}$$

```
[In] int(1/(5*cos(c + d*x) + 3)^4,x)
```

```
[Out] - (279*atanh(tan(c/2 + (d*x)/2)/2))/(16384*d) - ((295*tan(c/2 + (d*x)/2))/5
12 - (265*tan(c/2 + (d*x)/2)^3)/768 + (745*tan(c/2 + (d*x)/2)^5)/8192)/(d*(
48*tan(c/2 + (d*x)/2)^2 - 12*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 -
64))
```

3.38 $\int \frac{1}{3-5\cos(c+dx)}\,dx$

Optimal result 273
Rubi [A] (verified) 273
Mathematica [A] (verified) 274
Maple [A] (verified) 274
Fricas [A] (verification not implemented) 275
Sympy [A] (verification not implemented) 275
Maxima [A] (verification not implemented) 276
Giac [A] (verification not implemented) 276
Mupad [B] (verification not implemented) 276

Optimal result

Integrand size = 12, antiderivative size = 63

$$\int \frac{1}{3-5\cos(c+dx)}\,dx = \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[Out] 1/4*ln(cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c))/d-1/4*ln(cos(1/2*d*x+1/2*c)
+2*sin(1/2*d*x+1/2*c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2738, 213}

$$\int \frac{1}{3-5\cos(c+dx)}\,dx = \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[In] Int[(3 - 5*Cos[c + d*x])^(-1),x]
```

```
[Out] Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]]/(4*d) - Log[Cos[(c + d*x)/2] + 2
*Sin[(c + d*x)/2]]/(4*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\text{Subst}\left(\int \frac{1}{-2+8x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}\\ &= \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}-\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{1}{3-5\cos(c+dx)}\,dx = \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[In] Integrate[(3 - 5*Cos[c + d*x])^(-1),x]
[Out] Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]]/(4*d) - Log[Cos[(c + d*x)/2] + 2
*Sin[(c + d*x)/2]]/(4*d)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

method	result	size
parallelrisch	$\frac{-\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4d}$	37
derivativedivides	$\frac{-\frac{\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4}+\frac{\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4}}{d}$	38
default	$\frac{-\frac{\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4}+\frac{\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4}}{d}$	38
norman	$\frac{\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4d}-\frac{\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4d}$	40
risch	$\frac{\ln\left(e^{i(dx+c)}-\frac{3}{5}-\frac{4i}{5}\right)}{4d}-\frac{\ln\left(e^{i(dx+c)}-\frac{3}{5}+\frac{4i}{5}\right)}{4d}$	40

```
[In] int(1/(3-5*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-ln(2*tan(1/2*d*x+1/2*c)+1)+ln(2*tan(1/2*d*x+1/2*c)-1))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \frac{1}{3-5\cos(c+dx)}\,dx = -\frac{\log\left(-\frac{3}{2}\cos\left(dx+c\right)+2\sin\left(dx+c\right)+\frac{5}{2}\right)-\log\left(-\frac{3}{2}\cos\left(dx+c\right)-2\sin\left(dx+c\right)+\frac{5}{2}\right)}{8\,d}$$

```
[In] integrate(1/(3-5*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(log(-3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2) - log(-3/2*cos(d*x + c)
 - 2*sin(d*x + c) + 5/2))/d
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \frac{1}{3-5\cos(c+dx)}\,dx = \begin{cases} \frac{\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{4d}-\frac{\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{4d} & \text{for}\: d\neq 0 \\ \frac{x}{3-5\cos\left(c\right)} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(3-5*cos(d*x+c)),x)
```

```
[Out] Piecewise((log(2*tan(c/2 + d*x/2) - 1)/(4*d) - log(2*tan(c/2 + d*x/2) + 1)/
(4*d), Ne(d, 0)), (x/(3 - 5*cos(c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \frac{1}{3-5\cos(c+dx)}\,dx = -\frac{\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+1\right)-\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{4\,d}$$

```
[In] integrate(1/(3-5*cos(d*x+c)),x, algorithm="maxima")
[Out] -1/4*(log(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1) - log(2*sin(d*x + c)/(cos(
d*x + c) + 1) - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.60

$$\int \frac{1}{3-5\cos(c+dx)}\,dx = -\frac{\log\left(\left|2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\log\left(\left|2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{4\,d}$$

```
[In] integrate(1/(3-5*cos(d*x+c)),x, algorithm="giac")
[Out] -1/4*(log(abs(2*tan(1/2*d*x + 1/2*c) + 1)) - log(abs(2*tan(1/2*d*x + 1/2*c)
 - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.27

$$\int \frac{1}{3-5\cos(c+dx)}\,dx = -\frac{\mathrm{atanh}\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}$$

```
[In] int(-1/(5*cos(c + d*x) - 3),x)
[Out] -atanh(2*tan(c/2 + (d*x)/2))/(2*d)
```

3.39 $\int \frac{1}{(3-5\cos(c+dx))^2}\,dx$

Optimal result . . . 277
Rubi [A] (verified) . . . 277
Mathematica [A] (verified) . . . 279
Maple [A] (verified) . . . 279
Fricas [A] (verification not implemented) . . . 280
Sympy [B] (verification not implemented) . . . 280
Maxima [A] (verification not implemented) . . . 281
Giac [A] (verification not implemented) . . . 281
Mupad [B] (verification not implemented) . . . 281

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{1}{(3-5\cos(c+dx))^2}\,dx = -\frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{5\sin(c+dx)}{16d(3-5\cos(c+dx))}$$

```
[Out] -3/64*ln(cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c))/d+3/64*ln(cos(1/2*d*x+1/2
*c)+2*sin(1/2*d*x+1/2*c))/d-5/16*sin(d*x+c)/d/(3-5*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2743, 12, 2738, 213}

$$\int \frac{1}{(3-5\cos(c+dx))^2}\,dx = -\frac{5\sin(c+dx)}{16d(3-5\cos(c+dx))} - \frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{3\log\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{64d}$$

```
[In] Int[(3 - 5*Cos[c + d*x])^(-2),x]
```

```
[Out] (-3*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]])/(64*d) + (3*Log[Cos[(c + d*
x)/2] + 2*Sin[(c + d*x)/2]])/(64*d) - (5*Sin[c + d*x])/(16*d*(3 - 5*Cos[c +
 d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{5\sin(c+dx)}{16d(3-5\cos(c+dx))}+\frac{1}{16}\int -\frac{3}{3-5\cos(c+dx)}\,dx\\
&= -\frac{5\sin(c+dx)}{16d(3-5\cos(c+dx))}-\frac{3}{16}\int \frac{1}{3-5\cos(c+dx)}\,dx\\
&= -\frac{5\sin(c+dx)}{16d(3-5\cos(c+dx))}-\frac{3\text{Subst}\left(\int \frac{1}{-2+8x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{8d}\\
&= -\frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d}\\
&\quad +\frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d}-\frac{5\sin(c+dx)}{16d(3-5\cos(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.58

$$\int \frac{1}{(3-5\cos(c+dx))^2}\,dx$$
$$= \frac{9\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)-15\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)-\right.}{64d(-3+5\,\mathrm{c}}$$

```
[In] Integrate[(3 - 5*Cos[c + d*x])^(-2),x]
```

```
[Out] (9*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - 15*Cos[c + d*x]*(Log[Cos[(c
 + d*x)/2] - 2*Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2
]]) - 9*Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]] + 20*Sin[c + d*x])/(64*d
*(-3 + 5*Cos[c + d*x]))
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

method	result
norman	$-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\left(4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}-\frac{3\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{64d}+\frac{3\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{64d}$
derivativedivides	$\frac{-\frac{5}{64\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{3\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{64}-\frac{5}{64\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{64}}{d}$
default	$\frac{-\frac{5}{64\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{3\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{64}-\frac{5}{64\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{64}}{d}$
risch	$-\frac{i\left(3\,\mathrm{e}^{i(dx+c)}-5\right)}{8d\left(5\,\mathrm{e}^{2i(dx+c)}-6\,\mathrm{e}^{i(dx+c)}+5\right)}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-\frac{3}{5}+\frac{4i}{5}\right)}{64d}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-\frac{3}{5}-\frac{4i}{5}\right)}{64d}$
parallelrisch	$\frac{-15\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+15\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+20\sin(dx+c)+9\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-9\,\mathrm{l}}{64d(-3+5\cos(dx+c))}$

```
[In] int(1/(3-5*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -5/16/d*tan(1/2*d*x+1/2*c)/(4*tan(1/2*d*x+1/2*c)^2-1)-3/64/d*ln(2*tan(1/2*d
*x+1/2*c)-1)+3/64/d*ln(2*tan(1/2*d*x+1/2*c)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3-5\cos(c+dx))^2}\,dx = \frac{3\,(5\,\cos(dx+c)-3)\log\left(-\frac{3}{2}\,\cos(dx+c)+2\,\sin(dx+c)+\frac{5}{2}\right)-3\,(5\,\cos(dx+c)-3)\log\left(-\frac{3}{2}\,\cos(dx+\right.}{128\,(5\,d\cos(dx+c)-3\,d)}$$

```
[In] integrate(1/(3-5*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/128*(3*(5*cos(d*x + c) - 3)*log(-3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2)
 - 3*(5*cos(d*x + c) - 3)*log(-3/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 4
0*sin(d*x + c))/(5*d*cos(d*x + c) - 3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(78) = 156.

Time = 0.71 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.73

$$\int \frac{1}{(3-5\cos(c+dx))^2}\,dx = \begin{cases} \frac{x}{\left(3-5\cos\left(2\,\mathrm{atan}\left(\frac{1}{2}\right)\right)\right)^2} \\ \frac{x}{(3-5\cos(c))^2} \\ -\frac{12\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{256d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-64d}+\frac{3\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{256d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-64d}+\frac{12\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{256d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-64d}-\frac{3\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right.\right.}{256d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)} \end{cases}$$

```
[In] integrate(1/(3-5*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((x/(3 - 5*cos(2*atan(1/2)))**2, Eq(c, -d*x - 2*atan(1/2)) | Eq(c,
 -d*x + 2*atan(1/2))), (x/(3 - 5*cos(c))**2, Eq(d, 0)), (-12*log(2*tan(c/2
+ d*x/2) - 1)*tan(c/2 + d*x/2)**2/(256*d*tan(c/2 + d*x/2)**2 - 64*d) + 3*lo
g(2*tan(c/2 + d*x/2) - 1)/(256*d*tan(c/2 + d*x/2)**2 - 64*d) + 12*log(2*tan
(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(256*d*tan(c/2 + d*x/2)**2 - 64*d) -
 3*log(2*tan(c/2 + d*x/2) + 1)/(256*d*tan(c/2 + d*x/2)**2 - 64*d) - 20*tan(
c/2 + d*x/2)/(256*d*tan(c/2 + d*x/2)**2 - 64*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{1}{(3-5\cos(c+dx))^2}\,dx = -\frac{\frac{20\sin(dx+c)}{\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)} - 3\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+1\right) + 3\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{64\,d}$$

```
[In] integrate(1/(3-5*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/64*(20*sin(d*x + c)/((4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*
x + c) + 1)) - 3*log(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1) + 3*log(2*sin(d
*x + c)/(cos(d*x + c) + 1) - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{1}{(3-5\cos(c+dx))^2}\,dx = -\frac{\frac{20\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1} - 3\log\left(\left|2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) + 3\log\left(\left|2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{64\,d}$$

```
[In] integrate(1/(3-5*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/64*(20*tan(1/2*d*x + 1/2*c)/(4*tan(1/2*d*x + 1/2*c)^2 - 1) - 3*log(abs(2
*tan(1/2*d*x + 1/2*c) + 1)) + 3*log(abs(2*tan(1/2*d*x + 1/2*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{1}{(3-5\cos(c+dx))^2}\,dx = \frac{3\,\mathrm{atanh}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{32\,d} - \frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-\frac{1}{4}\right)}$$

```
[In] int(1/(5*cos(c + d*x) - 3)^2,x)
```

```
[Out] (3*atanh(2*tan(c/2 + (d*x)/2)))/(32*d) - (5*tan(c/2 + (d*x)/2))/(64*d*(tan(
c/2 + (d*x)/2)^2 - 1/4))
```

3.40 $\int \frac{1}{(3-5\cos(c+dx))^3}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 113

$$\int \frac{1}{(3-5\cos(c+dx))^3}\,dx = \frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{5\sin(c+dx)}{32d(3-5\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(3-5\cos(c+dx))}$$

```
[Out] 43/2048*ln(cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c))/d-43/2048*ln(cos(1/2*d*
x+1/2*c)+2*sin(1/2*d*x+1/2*c))/d-5/32*sin(d*x+c)/d/(3-5*cos(d*x+c))^2+45/51
2*sin(d*x+c)/d/(3-5*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2743, 2833, 12, 2738, 213}

$$\int \frac{1}{(3-5\cos(c+dx))^3}\,dx = \frac{45\sin(c+dx)}{512d(3-5\cos(c+dx))} - \frac{5\sin(c+dx)}{32d(3-5\cos(c+dx))^2} + \frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{43\log\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}$$

```
[In] Int[(3 - 5*Cos[c + d*x])^(-3),x]
```

```
[Out] (43*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]])/(2048*d) - (43*Log[Cos[(c +
 d*x)/2] + 2*Sin[(c + d*x)/2]])/(2048*d) - (5*Sin[c + d*x])/(32*d*(3 - 5*Co
s[c + d*x])^2) + (45*Sin[c + d*x])/(512*d*(3 - 5*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{5\sin(c+dx)}{32d(3-5\cos(c+dx))^2}+\frac{1}{32}\int\frac{-6-5\cos(c+dx)}{(3-5\cos(c+dx))^2}\,dx\\ &= -\frac{5\sin(c+dx)}{32d(3-5\cos(c+dx))^2}+\frac{45\sin(c+dx)}{512d(3-5\cos(c+dx))}+\frac{1}{512}\int\frac{43}{3-5\cos(c+dx)}\,dx\end{aligned}$$

$$= -\frac{5\sin(c+dx)}{32d(3-5\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(3-5\cos(c+dx))} + \frac{43}{512}\int \frac{1}{3-5\cos(c+dx)}\,dx$$

$$= -\frac{5\sin(c+dx)}{32d(3-5\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(3-5\cos(c+dx))} + \frac{43\text{Subst}\left(\int \frac{1}{-2+8x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{256d}$$

$$= \frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{5\sin(c+dx)}{32d(3-5\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(3-5\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.87

$$\int \frac{1}{(3-5\cos(c+dx))^3}\,dx = \frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{5}{512d\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{45\sin\left(\frac{1}{2}(c+dx)\right)}{1024d\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{5}{512d\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{45\sin\left(\frac{1}{2}(c+dx)\right)}{1024d\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

```
[In] Integrate[(3 - 5*Cos[c + d*x])^(-3),x]
```

```
[Out] (43*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]])/(2048*d) - (43*Log[Cos[(c +
 d*x)/2] + 2*Sin[(c + d*x)/2]])/(2048*d) - 5/(512*d*(Cos[(c + d*x)/2] - 2*S
in[(c + d*x)/2])^2) - (45*Sin[(c + d*x)/2])/(1024*d*(Cos[(c + d*x)/2] - 2*S
in[(c + d*x)/2])) + 5/(512*d*(Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2])^2) - (
45*Sin[(c + d*x)/2])/(1024*d*(Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]))
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

method	result
norman	$\frac{-\frac{85\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512d}+\frac{35\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d}}{\left(4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^2}+\frac{43\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2048d}-\frac{43\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2048d}$
derivativedivides	$\frac{-\frac{25}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{35}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{43\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2048}+\frac{25}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{35}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
default	$\frac{-\frac{25}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{35}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{43\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2048}+\frac{25}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{35}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
risch	$\frac{i\left(215\,\mathrm{e}^{3i(dx+c)}-387\,\mathrm{e}^{2i(dx+c)}+325\,\mathrm{e}^{i(dx+c)}-225\right)}{256d\left(5\,\mathrm{e}^{2i(dx+c)}-6\,\mathrm{e}^{i(dx+c)}+5\right)^2}+\frac{43\ln\left(\mathrm{e}^{i(dx+c)}-\frac{3}{5}-\frac{4i}{5}\right)}{2048d}-\frac{43\ln\left(\mathrm{e}^{i(dx+c)}-\frac{3}{5}+\frac{4i}{5}\right)}{2048d}$
parallelrisch	$\frac{(2580\cos(dx+c)-1075\cos(2dx+2c)-1849)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)+(-2580\cos(dx+c)+1075\cos(2dx+2c)+1849)\ln\left(\tan\right.}{2048d(-25\cos(2dx+2c)-43+60\cos(dx+c))}$

```
[In] int(1/(3-5*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-85/512/d*tan(1/2*d*x+1/2*c)+35/128/d*tan(1/2*d*x+1/2*c)^3)/(4*tan(1/2*d*x
+1/2*c)^2-1)^2+43/2048/d*ln(2*tan(1/2*d*x+1/2*c)-1)-43/2048/d*ln(2*tan(1/2*
d*x+1/2*c)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14

$$\int \frac{1}{(3-5\cos(c+dx))^3}\,dx = -\frac{43\left(25\cos\left(dx+c\right)^2-30\cos\left(dx+c\right)+9\right)\log\left(-\frac{3}{2}\cos\left(dx+c\right)+2\sin\left(dx+c\right)+\frac{5}{2}\right)-43\left(25\cos\left(dx\right.\right.}{4096\left(25\,d\cos\left(dx+\right.\right.}$$

```
[In] integrate(1/(3-5*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4096*(43*(25*cos(d*x + c)^2 - 30*cos(d*x + c) + 9)*log(-3/2*cos(d*x + c)
 + 2*sin(d*x + c) + 5/2) - 43*(25*cos(d*x + c)^2 - 30*cos(d*x + c) + 9)*log
(-3/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 40*(45*cos(d*x + c) - 11)*sin(
d*x + c))/(25*d*cos(d*x + c)^2 - 30*d*cos(d*x + c) + 9*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(102) = 204$.

Time = 1.19 (sec) , antiderivative size = 490, normalized size of antiderivative = 4.34

$$\int \frac{1}{(3-5\cos(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{x}{\left(3-5\cos\left(2\operatorname{atan}\left(\frac{1}{2}\right)\right)\right)^3} \\ \frac{x}{(3-5\cos(c))^3} \\ \frac{688\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{32768d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2048d} - \frac{344\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{32768d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2048d} + \frac{43\log\left(2\tan\left(\frac{c}{2}\cdots\right.\right.}{32768d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16384\cdots} \end{cases}$$

```
[In] integrate(1/(3-5*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((x/(3 - 5*cos(2*atan(1/2)))**3, Eq(c, -d*x - 2*atan(1/2)) | Eq(c,
 -d*x + 2*atan(1/2))), (x/(3 - 5*cos(c))**3, Eq(d, 0)), (688*log(2*tan(c/2
+ d*x/2) - 1)*tan(c/2 + d*x/2)**4/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*ta
n(c/2 + d*x/2)**2 + 2048*d) - 344*log(2*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x
/2)**2/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*d)
 + 43*log(2*tan(c/2 + d*x/2) - 1)/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*ta
n(c/2 + d*x/2)**2 + 2048*d) - 688*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x
/2)**4/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*d)
 + 344*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(32768*d*tan(c/2 + d
*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*d) - 43*log(2*tan(c/2 + d*x/2
) + 1)/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*d)
 + 560*tan(c/2 + d*x/2)**3/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 +
 d*x/2)**2 + 2048*d) - 340*tan(c/2 + d*x/2)/(32768*d*tan(c/2 + d*x/2)**4 -
16384*d*tan(c/2 + d*x/2)**2 + 2048*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

$$\int \frac{1}{(3-5\cos(c+dx))^3}\,dx$$
$$= \frac{\frac{20\left(\frac{17\sin(dx+c)}{\cos(dx+c)+1}-\frac{28\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{16\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-1} - 43\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+1\right) + 43\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{2048\,d}$$

```
[In] integrate(1/(3-5*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(20*(17*sin(d*x + c)/(cos(d*x + c) + 1) - 28*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 16*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 1) - 43*log(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1) + 4
3*log(2*sin(d*x + c)/(cos(d*x + c) + 1) - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \frac{1}{(3-5\cos(c+dx))^3}\,dx = \frac{\frac{20\left(28\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-17\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2} - 43\log\left(\left|2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) + 43\log\left(\left|2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{2048\,d}$$

```
[In] integrate(1/(3-5*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2048*(20*(28*tan(1/2*d*x + 1/2*c)^3 - 17*tan(1/2*d*x + 1/2*c))/(4*tan(1/2
*d*x + 1/2*c)^2 - 1)^2 - 43*log(abs(2*tan(1/2*d*x + 1/2*c) + 1)) + 43*log(a
bs(2*tan(1/2*d*x + 1/2*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int \frac{1}{(3-5\cos(c+dx))^3}\,dx = -\frac{43\,\mathrm{atanh}\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{1024\,d} - \frac{\frac{85\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8192} - \frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2048}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2} + \frac{1}{16}\right)}$$

```
[In] int(-1/(5*cos(c + d*x) - 3)^3,x)
```

```
[Out] - (43*atanh(2*tan(c/2 + (d*x)/2)))/(1024*d) - ((85*tan(c/2 + (d*x)/2))/8192
 - (35*tan(c/2 + (d*x)/2)^3)/2048)/(d*(tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*
x)/2)^2/2 + 1/16))
```

3.41 $\int \frac{1}{(3-5\cos(c+dx))^4} \, dx$

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Optimal result

Integrand size = 12, antiderivative size = 138

$$\int \frac{1}{(3-5\cos(c+dx))^4} \, dx = -\frac{279\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{279\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} - \frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3} + \frac{25\sin(c+dx)}{512d(3-5\cos(c+dx))^2} - \frac{995\sin(c+dx)}{24576d(3-5\cos(c+dx))}$$

```
[Out] -279/32768*ln(cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c))/d+279/32768*ln(cos(1
/2*d*x+1/2*c)+2*sin(1/2*d*x+1/2*c))/d-5/48*sin(d*x+c)/d/(3-5*cos(d*x+c))^3+
25/512*sin(d*x+c)/d/(3-5*cos(d*x+c))^2-995/24576*sin(d*x+c)/d/(3-5*cos(d*x+
c))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used

= {2743, 2833, 12, 2738, 213}

$$\int \frac{1}{(3-5\cos(c+dx))^4}\,dx = -\frac{995\sin(c+dx)}{24576d(3-5\cos(c+dx))} + \frac{25\sin(c+dx)}{512d(3-5\cos(c+dx))^2} - \frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3} - \frac{279\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{279\log\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}$$

```
[In] Int[(3 - 5*Cos[c + d*x])^(-4),x]
```

```
[Out] (-279*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]])/(32768*d) + (279*Log[Cos[
(c + d*x)/2] + 2*Sin[(c + d*x)/2]])/(32768*d) - (5*Sin[c + d*x])/(48*d*(3 -
 5*Cos[c + d*x])^3) + (25*Sin[c + d*x])/(512*d*(3 - 5*Cos[c + d*x])^2) - (9
95*Sin[c + d*x])/(24576*d*(3 - 5*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
```

```
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3}+\frac{1}{48}\int\frac{-9-10\cos(c+dx)}{(3-5\cos(c+dx))^3}\,dx\\
&= -\frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3}+\frac{25\sin(c+dx)}{512d(3-5\cos(c+dx))^2}+\frac{\int\frac{154+75\cos(c+dx)}{(3-5\cos(c+dx))^2}\,dx}{1536}\\
&= -\frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3}+\frac{25\sin(c+dx)}{512d(3-5\cos(c+dx))^2}\\
&\quad-\frac{995\sin(c+dx)}{24576d(3-5\cos(c+dx))}+\frac{\int-\frac{837}{3-5\cos(c+dx)}\,dx}{24576}\\
&= -\frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3}+\frac{25\sin(c+dx)}{512d(3-5\cos(c+dx))^2}\\
&\quad-\frac{995\sin(c+dx)}{24576d(3-5\cos(c+dx))}-\frac{279\int\frac{1}{3-5\cos(c+dx)}\,dx}{8192}\\
&= -\frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3}+\frac{25\sin(c+dx)}{512d(3-5\cos(c+dx))^2}\\
&\quad-\frac{995\sin(c+dx)}{24576d(3-5\cos(c+dx))}-\frac{279\text{Subst}\left(\int\frac{1}{-2+8x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{4096d}\\
&= -\frac{279\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}\\
&\quad+\frac{279\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}-\frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3}\\
&\quad+\frac{25\sin(c+dx)}{512d(3-5\cos(c+dx))^2}-\frac{995\sin(c+dx)}{24576d(3-5\cos(c+dx))}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 288 vs. $2(138) = 276$.

Time = 0.36 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.09

$$\int \frac{1}{(3-5\cos(c+dx))^4}\,dx$$
$$= \frac{467046\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)-104625\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(\right.\right.}{}$$

```
[In] Integrate[(3 - 5*Cos[c + d*x])^(-4),x]
```

```
[Out] (467046*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - 104625*Cos[3*(c + d*x)
]*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - 765855*Cos[c + d*x]*(Log[Cos
[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x
)/2]]) + 376650*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]
] - Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]]) - 467046*Log[Cos[(c + d*x)/
2] + 2*Sin[(c + d*x)/2]] + 104625*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + 2
*Sin[(c + d*x)/2]] + 226140*Sin[c + d*x] - 190800*Sin[2*(c + d*x)] + 99500*
Sin[3*(c + d*x)])/(393216*d*(-3 + 5*Cos[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.76

method	result
norman	$\frac{-\frac{745\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192d}+\frac{265\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768d}-\frac{295\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512d}}{\left(4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^3}-\frac{279\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{32768d}+\frac{279\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{32768d}$
risch	$-\frac{i\left(20925\,e^{5i(dx+c)}-62775\,e^{4i(dx+c)}+111042\,e^{3i(dx+c)}-119310\,e^{2i(dx+c)}+68625\,e^{i(dx+c)}-24875\right)}{12288d\left(5\,e^{2i(dx+c)}-6\,e^{i(dx+c)}+5\right)^3}+\frac{279\ln(e^{i(dx+c)}}{32768}$
derivativedivides	$\frac{-\frac{125}{49152\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{25}{8192\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{295}{32768\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{279\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{32768}-\frac{125}{49152\left(2\tan\left(\frac{dx}{2}\right.\right.}}{d}$
default	$\frac{-\frac{125}{49152\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{25}{8192\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{295}{32768\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{279\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{32768}-\frac{125}{49152\left(2\tan\left(\frac{dx}{2}\right.\right.}}{d}$
parallelrisch	$\frac{(-765855\cos(dx+c)+376650\cos(2dx+2c)-104625\cos(3dx+3c)+467046)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)+(765855\cos(dx+c)-3}{98304d(-558+125\cos(3d}$

```
[In] int(1/(3-5*cos(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-745/8192/d*tan(1/2*d*x+1/2*c)+265/768/d*tan(1/2*d*x+1/2*c)^3-295/512/d*ta
n(1/2*d*x+1/2*c)^5)/(4*tan(1/2*d*x+1/2*c)^2-1)^3-279/32768/d*ln(2*tan(1/2*d
*x+1/2*c)-1)+279/32768/d*ln(2*tan(1/2*d*x+1/2*c)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23

$$\int \frac{1}{(3-5\cos(c+dx))^4}\,dx$$
$$= \frac{837\left(125\,\cos\left(dx+c\right)^3-225\,\cos\left(dx+c\right)^2+135\,\cos\left(dx+c\right)-27\right)\log\left(-\frac{3}{2}\,\cos\left(dx+c\right)+2\,\sin\left(dx+c\right)\right.}{}$$

```
[In] integrate(1/(3-5*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/196608*(837*(125*cos(d*x + c)^3 - 225*cos(d*x + c)^2 + 135*cos(d*x + c) -
 27)*log(-3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2) - 837*(125*cos(d*x + c)^
3 - 225*cos(d*x + c)^2 + 135*cos(d*x + c) - 27)*log(-3/2*cos(d*x + c) - 2*s
in(d*x + c) + 5/2) + 40*(4975*cos(d*x + c)^2 - 4770*cos(d*x + c) + 1583)*si
n(d*x + c))/(125*d*cos(d*x + c)^3 - 225*d*cos(d*x + c)^2 + 135*d*cos(d*x +
c) - 27*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. 2(126) = 252.

Time = 2.47 (sec) , antiderivative size = 831, normalized size of antiderivative = 6.02

$$\int \frac{1}{(3-5\cos(c+dx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(3-5*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((x/(3 - 5*cos(2*atan(1/2)))**4, Eq(c, -d*x - 2*atan(1/2)) | Eq(c,
 -d*x + 2*atan(1/2))), (x/(3 - 5*cos(c))**4, Eq(d, 0)), (-53568*log(2*tan(c
/2 + d*x/2) - 1)*tan(c/2 + d*x/2)**6/(6291456*d*tan(c/2 + d*x/2)**6 - 47185
92*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d) + 40176
*log(2*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)**4/(6291456*d*tan(c/2 + d*x/2
)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 9830
4*d) - 10044*log(2*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)**2/(6291456*d*tan
(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/
2)**2 - 98304*d) + 837*log(2*tan(c/2 + d*x/2) - 1)/(6291456*d*tan(c/2 + d*x
/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98
304*d) + 53568*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**6/(6291456*d*t
an(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*
x/2)**2 - 98304*d) - 40176*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/
(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*
tan(c/2 + d*x/2)**2 - 98304*d) + 10044*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2
```

```
+ d*x/2)**2/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4
+ 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d) - 837*log(2*tan(c/2 + d*x/2) + 1
)/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*
d*tan(c/2 + d*x/2)**2 - 98304*d) - 56640*tan(c/2 + d*x/2)**5/(6291456*d*tan
(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/
2)**2 - 98304*d) + 33920*tan(c/2 + d*x/2)**3/(6291456*d*tan(c/2 + d*x/2)**6
 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d)
 - 8940*tan(c/2 + d*x/2)/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2
 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.28

$$\int \frac{1}{(3-5\cos(c+dx))^4}\,dx = -\frac{\frac{20\left(\frac{447\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{1696\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2832\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{12\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{48\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{64\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-1}-837\,\log\left(\frac{2\,\sin(dx+c)}{\cos(dx+c)+1}+1\right)+837\,\log\left(\frac{2\,\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{98304\,d}$$

```
[In] integrate(1/(3-5*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/98304*(20*(447*sin(d*x + c)/(cos(d*x + c) + 1) - 1696*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 2832*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(12*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 48*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 64*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 1) - 837*log(2*sin(d*x + c)/(cos(d*x +
c) + 1) + 1) + 837*log(2*sin(d*x + c)/(cos(d*x + c) + 1) - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.70

$$\int \frac{1}{(3-5\cos(c+dx))^4}\,dx = -\frac{\frac{20\left(2832\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1696\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+447\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}-837\,\log\left(\left|2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+837\,\log\left(\left|2\right.\right.}{98304\,d}$$

```
[In] integrate(1/(3-5*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/98304*(20*(2832*tan(1/2*d*x + 1/2*c)^5 - 1696*tan(1/2*d*x + 1/2*c)^3 + 4
47*tan(1/2*d*x + 1/2*c))/(4*tan(1/2*d*x + 1/2*c)^2 - 1)^3 - 837*log(abs(2*t
an(1/2*d*x + 1/2*c) + 1)) + 837*log(abs(2*tan(1/2*d*x + 1/2*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 15.70 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.74

$$\int \frac{1}{(3-5\cos(c+dx))^4}\,dx = \frac{279\,\mathrm{atanh}\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{16384\,d} - \frac{\frac{295\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{32768} - \frac{265\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{49152} + \frac{745\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{524288}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{4} + \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{16} - \frac{1}{64}\right)}$$

```
[In] int(1/(5*cos(c + d*x) - 3)^4,x)
[Out] (279*atanh(2*tan(c/2 + (d*x)/2)))/(16384*d) - ((745*tan(c/2 + (d*x)/2))/524
288 - (265*tan(c/2 + (d*x)/2)^3)/49152 + (295*tan(c/2 + (d*x)/2)^5)/32768)/
(d*((3*tan(c/2 + (d*x)/2)^2)/16 - (3*tan(c/2 + (d*x)/2)^4)/4 + tan(c/2 + (d
*x)/2)^6 - 1/64))
```

3.42 $\int \frac{1}{-3+5\cos(c+dx)}\,dx$

Optimal result 295
Rubi [A] (verified) 295
Mathematica [A] (verified) 296
Maple [A] (verified) 296
Fricas [A] (verification not implemented) 297
Sympy [A] (verification not implemented) 297
Maxima [A] (verification not implemented) 298
Giac [A] (verification not implemented) 298
Mupad [B] (verification not implemented) 298

Optimal result

Integrand size = 12, antiderivative size = 63

$$\int \frac{1}{-3+5\cos(c+dx)}\,dx = -\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[Out] -1/4*ln(cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c))/d+1/4*ln(cos(1/2*d*x+1/2*c
)+2*sin(1/2*d*x+1/2*c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2738, 212}

$$\int \frac{1}{-3+5\cos(c+dx)}\,dx = \frac{\log\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[In] Int[(-3 + 5*Cos[c + d*x])^(-1),x]
```

```
[Out] -1/4*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]]/d + Log[Cos[(c + d*x)/2] +
2*Sin[(c + d*x)/2]]/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{1}{2-8x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$
$$= -\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{1}{-3+5\cos(c+dx)}\,dx = -\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[In] Integrate[(-3 + 5*Cos[c + d*x])^(-1),x]
```

```
[Out] -1/4*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]]/d + Log[Cos[(c + d*x)/2] +
2*Sin[(c + d*x)/2]]/(4*d)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

method	result	size
parallelrisch	$\frac{\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4d}$	37
derivativedivides	$\frac{\frac{\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4}-\frac{\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4}}{d}$	38
default	$\frac{\frac{\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4}-\frac{\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4}}{d}$	38
norman	$-\frac{\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4d}+\frac{\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4d}$	40
risch	$-\frac{\ln\left(e^{i(dx+c)}-\frac{3}{5}-\frac{4i}{5}\right)}{4d}+\frac{\ln\left(e^{i(dx+c)}-\frac{3}{5}+\frac{4i}{5}\right)}{4d}$	40

```
[In] int(1/(-3+5*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(ln(2*tan(1/2*d*x+1/2*c)+1)-ln(2*tan(1/2*d*x+1/2*c)-1))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \frac{1}{-3+5\cos(c+dx)}\,dx = \frac{\log\left(-\frac{3}{2}\cos\left(dx+c\right)+2\sin\left(dx+c\right)+\frac{5}{2}\right)-\log\left(-\frac{3}{2}\cos\left(dx+c\right)-2\sin\left(dx+c\right)+\frac{5}{2}\right)}{8\,d}$$

```
[In] integrate(1/(-3+5*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(log(-3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2) - log(-3/2*cos(d*x + c)
- 2*sin(d*x + c) + 5/2))/d
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \frac{1}{-3+5\cos(c+dx)}\,dx = \begin{cases} -\frac{\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{4d}+\frac{\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{4d} & \text{for}\ d\neq 0 \\ \frac{x}{5\cos\left(c\right)-3} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(-3+5*cos(d*x+c)),x)
```

```
[Out] Piecewise((-log(2*tan(c/2 + d*x/2) - 1)/(4*d) + log(2*tan(c/2 + d*x/2) + 1)
/(4*d), Ne(d, 0)), (x/(5*cos(c) - 3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \frac{1}{-3+5\cos(c+dx)}\,dx = \frac{\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+1\right)-\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{4\,d}$$

```
[In] integrate(1/(-3+5*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(log(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1) - log(2*sin(d*x + c)/(cos(d
*x + c) + 1) - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.60

$$\int \frac{1}{-3+5\cos(c+dx)}\,dx = \frac{\log\left(\left|2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\log\left(\left|2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{4\,d}$$

```
[In] integrate(1/(-3+5*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(2*tan(1/2*d*x + 1/2*c) + 1)) - log(abs(2*tan(1/2*d*x + 1/2*c)
- 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.27

$$\int \frac{1}{-3+5\cos(c+dx)}\,dx = \frac{\mathrm{atanh}\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}$$

```
[In] int(1/(5*cos(c + d*x) - 3),x)
```

```
[Out] atanh(2*tan(c/2 + (d*x)/2))/(2*d)
```

3.43 $\int \frac{1}{(-3+5\cos(c+dx))^2}\,dx$

Optimal result 299
Rubi [A] (verified) 299
Mathematica [A] (verified) 301
Maple [A] (verified) 301
Fricas [A] (verification not implemented) 302
Sympy [B] (verification not implemented) 302
Maxima [A] (verification not implemented) 303
Giac [A] (verification not implemented) 303
Mupad [B] (verification not implemented) 303

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{1}{(-3+5\cos(c+dx))^2}\,dx = -\frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{5\sin(c+dx)}{16d(3-5\cos(c+dx))}$$

```
[Out] -3/64*ln(cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c))/d+3/64*ln(cos(1/2*d*x+1/2
*c)+2*sin(1/2*d*x+1/2*c))/d-5/16*sin(d*x+c)/d/(3-5*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2743, 12, 2738, 212}

$$\int \frac{1}{(-3+5\cos(c+dx))^2}\,dx = -\frac{5\sin(c+dx)}{16d(3-5\cos(c+dx))} - \frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{3\log\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{64d}$$

```
[In] Int[(-3 + 5*Cos[c + d*x])^(-2),x]
```

```
[Out] (-3*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]])/(64*d) + (3*Log[Cos[(c + d*
x)/2] + 2*Sin[(c + d*x)/2]])/(64*d) - (5*Sin[c + d*x])/(16*d*(3 - 5*Cos[c +
 d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{5\sin(c+dx)}{16d(3-5\cos(c+dx))}+\frac{1}{16}\int\frac{3}{-3+5\cos(c+dx)}\,dx\\
&= -\frac{5\sin(c+dx)}{16d(3-5\cos(c+dx))}+\frac{3}{16}\int\frac{1}{-3+5\cos(c+dx)}\,dx\\
&= -\frac{5\sin(c+dx)}{16d(3-5\cos(c+dx))}+\frac{3\text{Subst}\left(\int\frac{1}{2-8x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{8d}\\
&= -\frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d}\\
&\quad+\frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d}-\frac{5\sin(c+dx)}{16d(3-5\cos(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.58

$$\int \frac{1}{(-3+5\cos(c+dx))^2}\,dx = \frac{9\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)-15\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)-\right.}{64d(-3+5\,\mathrm{c}}$$

```
[In] Integrate[(-3 + 5*Cos[c + d*x])^(-2),x]
```

```
[Out] (9*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - 15*Cos[c + d*x]*(Log[Cos[(c
 + d*x)/2] - 2*Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2
]]) - 9*Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]] + 20*Sin[c + d*x])/(64*d
*(-3 + 5*Cos[c + d*x]))
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

method	result
norman	$-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\left(4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}-\frac{3\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{64d}+\frac{3\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{64d}$
derivativedivides	$\frac{-\frac{5}{64\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{3\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{64}-\frac{5}{64\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{64}}{d}$
default	$\frac{-\frac{5}{64\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{3\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{64}-\frac{5}{64\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{64}}{d}$
risch	$-\frac{i\left(3\,\mathrm{e}^{i(dx+c)}-5\right)}{8d\left(5\,\mathrm{e}^{2i(dx+c)}-6\,\mathrm{e}^{i(dx+c)}+5\right)}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-\frac{3}{5}+\frac{4i}{5}\right)}{64d}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-\frac{3}{5}-\frac{4i}{5}\right)}{64d}$
parallelrisch	$\frac{-15\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+15\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+20\sin(dx+c)+9\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-9\,\mathrm{l}}{64d(-3+5\cos(dx+c))}$

```
[In] int(1/(-3+5*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -5/16/d*tan(1/2*d*x+1/2*c)/(4*tan(1/2*d*x+1/2*c)^2-1)-3/64/d*ln(2*tan(1/2*d
*x+1/2*c)-1)+3/64/d*ln(2*tan(1/2*d*x+1/2*c)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-3+5\cos(c+dx))^2}\,dx = \frac{3\,(5\,\cos(dx+c)-3)\log\left(-\frac{3}{2}\,\cos(dx+c)+2\,\sin(dx+c)+\frac{5}{2}\right)-3\,(5\,\cos(dx+c)-3)\log\left(-\frac{3}{2}\,\cos(dx+\right.}{128\,(5\,d\cos(dx+c)-3\,d)}$$

```
[In] integrate(1/(-3+5*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/128*(3*(5*cos(d*x + c) - 3)*log(-3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2)
 - 3*(5*cos(d*x + c) - 3)*log(-3/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 4
0*sin(d*x + c))/(5*d*cos(d*x + c) - 3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(78) = 156.

Time = 0.68 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.73

$$\int \frac{1}{(-3+5\cos(c+dx))^2}\,dx = \begin{cases} \frac{x}{\left(-3+5\cos\left(2\,\mathrm{atan}\left(\frac{1}{2}\right)\right)\right)^2} \\ \frac{x}{(5\cos(c)-3)^2} \\ -\frac{12\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{256d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-64d}+\frac{3\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{256d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-64d}+\frac{12\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{256d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-64d}-\frac{3\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right.\right.}{256d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)} \end{cases}$$

```
[In] integrate(1/(-3+5*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((x/(-3 + 5*cos(2*atan(1/2)))**2, Eq(c, -d*x - 2*atan(1/2)) | Eq(c
, -d*x + 2*atan(1/2))), (x/(5*cos(c) - 3)**2, Eq(d, 0)), (-12*log(2*tan(c/2
 + d*x/2) - 1)*tan(c/2 + d*x/2)**2/(256*d*tan(c/2 + d*x/2)**2 - 64*d) + 3*l
og(2*tan(c/2 + d*x/2) - 1)/(256*d*tan(c/2 + d*x/2)**2 - 64*d) + 12*log(2*ta
n(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(256*d*tan(c/2 + d*x/2)**2 - 64*d)
- 3*log(2*tan(c/2 + d*x/2) + 1)/(256*d*tan(c/2 + d*x/2)**2 - 64*d) - 20*tan
(c/2 + d*x/2)/(256*d*tan(c/2 + d*x/2)**2 - 64*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{1}{(-3+5\cos(c+dx))^2}\,dx = -\frac{\frac{20\sin(dx+c)}{\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)} - 3\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+1\right) + 3\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{64\,d}$$

```
[In] integrate(1/(-3+5*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/64*(20*sin(d*x + c)/((4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*
x + c) + 1)) - 3*log(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1) + 3*log(2*sin(d
*x + c)/(cos(d*x + c) + 1) - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{1}{(-3+5\cos(c+dx))^2}\,dx = -\frac{\frac{20\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1} - 3\log\left(\left|2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) + 3\log\left(\left|2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{64\,d}$$

```
[In] integrate(1/(-3+5*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/64*(20*tan(1/2*d*x + 1/2*c)/(4*tan(1/2*d*x + 1/2*c)^2 - 1) - 3*log(abs(2
*tan(1/2*d*x + 1/2*c) + 1)) + 3*log(abs(2*tan(1/2*d*x + 1/2*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{1}{(-3+5\cos(c+dx))^2}\,dx = \frac{3\,\mathrm{atanh}\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{32\,d} - \frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{1}{4}\right)}$$

```
[In] int(1/(5*cos(c + d*x) - 3)^2,x)
```

```
[Out] (3*atanh(2*tan(c/2 + (d*x)/2)))/(32*d) - (5*tan(c/2 + (d*x)/2))/(64*d*(tan(
c/2 + (d*x)/2)^2 - 1/4))
```

3.44 $\int \frac{1}{(-3+5\cos(c+dx))^3}\,dx$

Optimal result . . . 304
Rubi [A] (verified) . . . 304
Mathematica [A] (verified) . . . 306
Maple [A] (verified) . . . 307
Fricas [A] (verification not implemented) . . . 307
Sympy [B] (verification not implemented) . . . 308
Maxima [A] (verification not implemented) . . . 308
Giac [A] (verification not implemented) . . . 309
Mupad [B] (verification not implemented) . . . 309

Optimal result

Integrand size = 12, antiderivative size = 113

$$\int \frac{1}{(-3+5\cos(c+dx))^3}\,dx = -\frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{5\sin(c+dx)}{32d(3-5\cos(c+dx))^2} - \frac{45\sin(c+dx)}{512d(3-5\cos(c+dx))}$$

```
[Out] -43/2048*ln(cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c))/d+43/2048*ln(cos(1/2*d
*x+1/2*c)+2*sin(1/2*d*x+1/2*c))/d+5/32*sin(d*x+c)/d/(3-5*cos(d*x+c))^2-45/5
12*sin(d*x+c)/d/(3-5*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2743, 2833, 12, 2738, 212}

$$\int \frac{1}{(-3+5\cos(c+dx))^3}\,dx = -\frac{45\sin(c+dx)}{512d(3-5\cos(c+dx))} + \frac{5\sin(c+dx)}{32d(3-5\cos(c+dx))^2} - \frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{43\log\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}$$

```
[In] Int[(-3 + 5*Cos[c + d*x])^(-3),x]
```

```
[Out] (-43*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]])/(2048*d) + (43*Log[Cos[(c
+ d*x)/2] + 2*Sin[(c + d*x)/2]])/(2048*d) + (5*Sin[c + d*x])/(32*d*(3 - 5*C
os[c + d*x])^2) - (45*Sin[c + d*x])/(512*d*(3 - 5*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{5\sin(c+dx)}{32d(3-5\cos(c+dx))^2}+\frac{1}{32}\int\frac{6+5\cos(c+dx)}{(-3+5\cos(c+dx))^2}\,dx\\ &= \frac{5\sin(c+dx)}{32d(3-5\cos(c+dx))^2}-\frac{45\sin(c+dx)}{512d(3-5\cos(c+dx))}+\frac{1}{512}\int\frac{43}{-3+5\cos(c+dx)}\,dx\end{aligned}$$

$$
\begin{aligned}
&= \frac{5\sin(c+dx)}{32d(3-5\cos(c+dx))^2} - \frac{45\sin(c+dx)}{512d(3-5\cos(c+dx))} + \frac{43}{512}\int \frac{1}{-3+5\cos(c+dx)}\,dx \\
&= \frac{5\sin(c+dx)}{32d(3-5\cos(c+dx))^2} - \frac{45\sin(c+dx)}{512d(3-5\cos(c+dx))} + \frac{43\mathrm{Subst}\left(\int \frac{1}{2-8x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{256d} \\
&= -\frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} \\
&\quad + \frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} \\
&\quad + \frac{5\sin(c+dx)}{32d(3-5\cos(c+dx))^2} - \frac{45\sin(c+dx)}{512d(3-5\cos(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.87

$$
\begin{aligned}
\int \frac{1}{(-3+5\cos(c+dx))^3}\,dx &= -\frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} \\
&\quad + \frac{43\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} \\
&\quad + \frac{5}{512d\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \\
&\quad + \frac{45\sin\left(\frac{1}{2}(c+dx)\right)}{1024d\left(\cos\left(\frac{1}{2}(c+dx)\right) - 2\sin\left(\frac{1}{2}(c+dx)\right)\right)} \\
&\quad - \frac{5}{512d\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \\
&\quad + \frac{45\sin\left(\frac{1}{2}(c+dx)\right)}{1024d\left(\cos\left(\frac{1}{2}(c+dx)\right) + 2\sin\left(\frac{1}{2}(c+dx)\right)\right)}
\end{aligned}
$$

```
[In] Integrate[(-3 + 5*Cos[c + d*x])^(-3),x]
```

```
[Out] (-43*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]])/(2048*d) + (43*Log[Cos[(c
+ d*x)/2] + 2*Sin[(c + d*x)/2]])/(2048*d) + 5/(512*d*(Cos[(c + d*x)/2] - 2*
Sin[(c + d*x)/2])^2) + (45*Sin[(c + d*x)/2])/(1024*d*(Cos[(c + d*x)/2] - 2*
Sin[(c + d*x)/2])) - 5/(512*d*(Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2])^2) +
(45*Sin[(c + d*x)/2])/(1024*d*(Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]))
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

method	result
norman	$\frac{\frac{85\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512d}-\frac{35\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d}}{\left(4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^2}-\frac{43\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2048d}+\frac{43\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2048d}$
derivativedivides	$\frac{-\frac{25}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{35}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{43\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2048}+\frac{25}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{35}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
default	$\frac{-\frac{25}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{35}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{43\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2048}+\frac{25}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{35}{2048\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
risch	$-\frac{i\left(215\,e^{3i(dx+c)}-387\,e^{2i(dx+c)}+325\,e^{i(dx+c)}-225\right)}{256d\left(5\,e^{2i(dx+c)}-6\,e^{i(dx+c)}+5\right)^2}+\frac{43\ln\left(e^{i(dx+c)}-\frac{3}{5}+\frac{4i}{5}\right)}{2048d}-\frac{43\ln\left(e^{i(dx+c)}-\frac{3}{5}-\frac{4i}{5}\right)}{2048d}$
parallelrisch	$\frac{(-2580\cos(dx+c)+1075\cos(2dx+2c)+1849)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)+(2580\cos(dx+c)-1075\cos(2dx+2c)-1849)\ln\left(\tan\right.}{2048d(-25\cos(2dx+2c)-43+60\cos(dx+c))}$

```
[In] int(1/(-3+5*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (85/512/d*tan(1/2*d*x+1/2*c)-35/128/d*tan(1/2*d*x+1/2*c)^3)/(4*tan(1/2*d*x+
1/2*c)^2-1)^2-43/2048/d*ln(2*tan(1/2*d*x+1/2*c)-1)+43/2048/d*ln(2*tan(1/2*d
*x+1/2*c)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14

$$\int \frac{1}{(-3+5\cos(c+dx))^3}\,dx = \frac{43\left(25\cos\left(dx+c\right)^2-30\cos\left(dx+c\right)+9\right)\log\left(-\frac{3}{2}\cos\left(dx+c\right)+2\sin\left(dx+c\right)+\frac{5}{2}\right)-43\left(25\cos\left(dx+\right.\right.}{4096\left(25\,d\cos\left(dx+c\right)\right.}$$

```
[In] integrate(1/(-3+5*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4096*(43*(25*cos(d*x + c)^2 - 30*cos(d*x + c) + 9)*log(-3/2*cos(d*x + c)
+ 2*sin(d*x + c) + 5/2) - 43*(25*cos(d*x + c)^2 - 30*cos(d*x + c) + 9)*log(
-3/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 40*(45*cos(d*x + c) - 11)*sin(d
*x + c))/(25*d*cos(d*x + c)^2 - 30*d*cos(d*x + c) + 9*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(102) = 204$.

Time = 1.20 (sec) , antiderivative size = 490, normalized size of antiderivative = 4.34

$$\int \frac{1}{(-3+5\cos(c+dx))^3}\,dx = \begin{cases} \frac{x}{\left(-3+5\cos\left(2\operatorname{atan}\left(\frac{1}{2}\right)\right)\right)^3} \\ \frac{x}{(5\cos(c)-3)^3} \\ -\frac{688\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{32768d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2048d} + \frac{344\log\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{32768d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2048d} - \frac{43\log\left(2\tan\right.}{32768d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16} \end{cases}$$

```
[In] integrate(1/(-3+5*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((x/(-3 + 5*cos(2*atan(1/2)))**3, Eq(c, -d*x - 2*atan(1/2)) | Eq(c
, -d*x + 2*atan(1/2))), (x/(5*cos(c) - 3)**3, Eq(d, 0)), (-688*log(2*tan(c/
2 + d*x/2) - 1)*tan(c/2 + d*x/2)**4/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*
tan(c/2 + d*x/2)**2 + 2048*d) + 344*log(2*tan(c/2 + d*x/2) - 1)*tan(c/2 + d
*x/2)**2/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*
d) - 43*log(2*tan(c/2 + d*x/2) - 1)/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*
tan(c/2 + d*x/2)**2 + 2048*d) + 688*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2 + d
*x/2)**4/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*
d) - 344*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(32768*d*tan(c/2 +
 d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*d) + 43*log(2*tan(c/2 + d*x
/2) + 1)/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 2048*
d) - 560*tan(c/2 + d*x/2)**3/(32768*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2
 + d*x/2)**2 + 2048*d) + 340*tan(c/2 + d*x/2)/(32768*d*tan(c/2 + d*x/2)**4
- 16384*d*tan(c/2 + d*x/2)**2 + 2048*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

$$\int \frac{1}{(-3+5\cos(c+dx))^3}\,dx = -\frac{\frac{20\left(\frac{17\sin(dx+c)}{\cos(dx+c)+1}-\frac{28\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{16\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-1} - 43\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+1\right) + 43\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{2048\,d}$$

```
[In] integrate(1/(-3+5*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2048*(20*(17*sin(d*x + c)/(cos(d*x + c) + 1) - 28*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 16*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 1) - 43*log(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1) +
43*log(2*sin(d*x + c)/(cos(d*x + c) + 1) - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-3+5\cos(c+dx))^3}\,dx = -\frac{\frac{20\left(28\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-17\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2} - 43\log\left(\left|2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 43\log\left(\left|2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{2048\,d}$$

```
[In] integrate(1/(-3+5*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2048*(20*(28*tan(1/2*d*x + 1/2*c)^3 - 17*tan(1/2*d*x + 1/2*c))/(4*tan(1/
2*d*x + 1/2*c)^2 - 1)^2 - 43*log(abs(2*tan(1/2*d*x + 1/2*c) + 1)) + 43*log(
abs(2*tan(1/2*d*x + 1/2*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int \frac{1}{(-3+5\cos(c+dx))^3}\,dx = \frac{43\,\mathrm{atanh}\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{1024\,d} + \frac{\frac{85\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8192}-\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2048}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\frac{1}{16}\right)}$$

```
[In] int(1/(5*cos(c + d*x) - 3)^3,x)
```

```
[Out] (43*atanh(2*tan(c/2 + (d*x)/2)))/(1024*d) + ((85*tan(c/2 + (d*x)/2))/8192 -
 (35*tan(c/2 + (d*x)/2)^3)/2048)/(d*(tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)
/2)^2/2 + 1/16))
```

3.45 $\int \frac{1}{(-3+5\cos(c+dx))^4}\,dx$

Optimal result 310
Rubi [A] (verified) 310
Mathematica [B] (verified) 313
Maple [A] (verified) 313
Fricas [A] (verification not implemented) 314
Sympy [B] (verification not implemented) 314
Maxima [A] (verification not implemented) 315
Giac [A] (verification not implemented) 315
Mupad [B] (verification not implemented) 316

Optimal result

Integrand size = 12, antiderivative size = 138

$$\int \frac{1}{(-3+5\cos(c+dx))^4}\,dx = -\frac{279\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{279\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} - \frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3} + \frac{25\sin(c+dx)}{512d(3-5\cos(c+dx))^2} - \frac{995\sin(c+dx)}{24576d(3-5\cos(c+dx))}$$

```
[Out] -279/32768*ln(cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c))/d+279/32768*ln(cos(1
/2*d*x+1/2*c)+2*sin(1/2*d*x+1/2*c))/d-5/48*sin(d*x+c)/d/(3-5*cos(d*x+c))^3+
25/512*sin(d*x+c)/d/(3-5*cos(d*x+c))^2-995/24576*sin(d*x+c)/d/(3-5*cos(d*x+
c))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used

$= \{2743, 2833, 12, 2738, 212\}$

$$\int \frac{1}{(-3+5\cos(c+dx))^4}\,dx = -\frac{995\sin(c+dx)}{24576d(3-5\cos(c+dx))} + \frac{25\sin(c+dx)}{512d(3-5\cos(c+dx))^2} - \frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3} - \frac{279\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{279\log\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}$$

```
[In] Int[(-3 + 5*Cos[c + d*x])^(-4),x]
```

```
[Out] (-279*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]])/(32768*d) + (279*Log[Cos[
(c + d*x)/2] + 2*Sin[(c + d*x)/2]])/(32768*d) - (5*Sin[c + d*x])/(48*d*(3 -
 5*Cos[c + d*x])^3) + (25*Sin[c + d*x])/(512*d*(3 - 5*Cos[c + d*x])^2) - (9
95*Sin[c + d*x])/(24576*d*(3 - 5*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
```

```
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3}+\frac{1}{48}\int \frac{9+10\cos(c+dx)}{(-3+5\cos(c+dx))^3}\,dx\\
&= -\frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3}+\frac{25\sin(c+dx)}{512d(3-5\cos(c+dx))^2}+\frac{\int \frac{154+75\cos(c+dx)}{(-3+5\cos(c+dx))^2}\,dx}{1536}\\
&= -\frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3}+\frac{25\sin(c+dx)}{512d(3-5\cos(c+dx))^2}\\
&\quad -\frac{995\sin(c+dx)}{24576d(3-5\cos(c+dx))}+\frac{\int \frac{837}{-3+5\cos(c+dx)}\,dx}{24576}\\
&= -\frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3}+\frac{25\sin(c+dx)}{512d(3-5\cos(c+dx))^2}\\
&\quad -\frac{995\sin(c+dx)}{24576d(3-5\cos(c+dx))}+\frac{279\int \frac{1}{-3+5\cos(c+dx)}\,dx}{8192}\\
&= -\frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3}+\frac{25\sin(c+dx)}{512d(3-5\cos(c+dx))^2}\\
&\quad -\frac{995\sin(c+dx)}{24576d(3-5\cos(c+dx))}+\frac{279\text{Subst}\left(\int \frac{1}{2-8x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{4096d}\\
&= -\frac{279\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}\\
&\quad +\frac{279\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}-\frac{5\sin(c+dx)}{48d(3-5\cos(c+dx))^3}\\
&\quad +\frac{25\sin(c+dx)}{512d(3-5\cos(c+dx))^2}-\frac{995\sin(c+dx)}{24576d(3-5\cos(c+dx))}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 288 vs. $2(138) = 276$.

Time = 0.01 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.09

$$\int \frac{1}{(-3+5\cos(c+dx))^4}\,dx$$
$$= \frac{467046\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)-104625\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{1}{2}(\right.\right.}{}$$

```
[In] Integrate[(-3 + 5*Cos[c + d*x])^(-4),x]
```

```
[Out] (467046*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - 104625*Cos[3*(c + d*x)
]*Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - 765855*Cos[c + d*x]*(Log[Cos
[(c + d*x)/2] - 2*Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x
)/2]]) + 376650*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - 2*Sin[(c + d*x)/2]
] - Log[Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2]]) - 467046*Log[Cos[(c + d*x)/
2] + 2*Sin[(c + d*x)/2]] + 104625*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + 2
*Sin[(c + d*x)/2]] + 226140*Sin[c + d*x] - 190800*Sin[2*(c + d*x)] + 99500*
Sin[3*(c + d*x)])/(393216*d*(-3 + 5*Cos[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.76

method	result
norman	$\frac{-\frac{745\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192d}+\frac{265\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768d}-\frac{295\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512d}}{\left(4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^3}-\frac{279\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{32768d}+\frac{279\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{32768d}$
risch	$-\frac{i\left(20925\,e^{5i(dx+c)}-62775\,e^{4i(dx+c)}+111042\,e^{3i(dx+c)}-119310\,e^{2i(dx+c)}+68625\,e^{i(dx+c)}-24875\right)}{12288d\left(5\,e^{2i(dx+c)}-6\,e^{i(dx+c)}+5\right)^3}+\frac{279\ln(e^{i(dx+c)}}{32768}$
derivativedivides	$\frac{-\frac{125}{49152\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{25}{8192\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{295}{32768\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{279\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{32768}-\frac{125}{49152\left(2\tan\left(\frac{dx}{2}\right.\right.}}{d}$
default	$\frac{-\frac{125}{49152\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{25}{8192\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{295}{32768\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{279\ln\left(2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{32768}-\frac{125}{49152\left(2\tan\left(\frac{dx}{2}\right.\right.}}{d}$
parallelrisch	$\frac{(-765855\cos(dx+c)+376650\cos(2dx+2c)-104625\cos(3dx+3c)+467046)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)+(765855\cos(dx+c)-3}{98304d(-558+125\cos(3dx}$

```
[In] int(1/(-3+5*cos(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-745/8192/d*tan(1/2*d*x+1/2*c)+265/768/d*tan(1/2*d*x+1/2*c)^3-295/512/d*ta
n(1/2*d*x+1/2*c)^5)/(4*tan(1/2*d*x+1/2*c)^2-1)^3-279/32768/d*ln(2*tan(1/2*d
*x+1/2*c)-1)+279/32768/d*ln(2*tan(1/2*d*x+1/2*c)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23

$$\int \frac{1}{(-3+5\cos(c+dx))^4}\,dx$$
$$= \frac{837\left(125\cos\left(dx+c\right)^3-225\cos\left(dx+c\right)^2+135\cos\left(dx+c\right)-27\right)\log\left(-\frac{3}{2}\cos\left(dx+c\right)+2\sin\left(dx+c\right)\right.}{}$$

```
[In] integrate(1/(-3+5*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/196608*(837*(125*cos(d*x + c)^3 - 225*cos(d*x + c)^2 + 135*cos(d*x + c) -
 27)*log(-3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2) - 837*(125*cos(d*x + c)^
3 - 225*cos(d*x + c)^2 + 135*cos(d*x + c) - 27)*log(-3/2*cos(d*x + c) - 2*s
in(d*x + c) + 5/2) + 40*(4975*cos(d*x + c)^2 - 4770*cos(d*x + c) + 1583)*si
n(d*x + c))/(125*d*cos(d*x + c)^3 - 225*d*cos(d*x + c)^2 + 135*d*cos(d*x +
c) - 27*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. 2(126) = 252.

Time = 2.48 (sec) , antiderivative size = 831, normalized size of antiderivative = 6.02

$$\int \frac{1}{(-3+5\cos(c+dx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-3+5*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((x/(-3 + 5*cos(2*atan(1/2)))**4, Eq(c, -d*x - 2*atan(1/2)) | Eq(c
, -d*x + 2*atan(1/2))), (x/(5*cos(c) - 3)**4, Eq(d, 0)), (-53568*log(2*tan(
c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)**6/(6291456*d*tan(c/2 + d*x/2)**6 - 4718
592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d) + 4017
6*log(2*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)**4/(6291456*d*tan(c/2 + d*x/
2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 983
04*d) - 10044*log(2*tan(c/2 + d*x/2) - 1)*tan(c/2 + d*x/2)**2/(6291456*d*ta
n(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x
/2)**2 - 98304*d) + 837*log(2*tan(c/2 + d*x/2) - 1)/(6291456*d*tan(c/2 + d*
x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 9
8304*d) + 53568*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**6/(6291456*d*
tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d
*x/2)**2 - 98304*d) - 40176*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4
/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d
*tan(c/2 + d*x/2)**2 - 98304*d) + 10044*log(2*tan(c/2 + d*x/2) + 1)*tan(c/2
```

```
 + d*x/2)**2/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4
 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d) - 837*log(2*tan(c/2 + d*x/2) +
1)/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648
*d*tan(c/2 + d*x/2)**2 - 98304*d) - 56640*tan(c/2 + d*x/2)**5/(6291456*d*ta
n(c/2 + d*x/2)**6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x
/2)**2 - 98304*d) + 33920*tan(c/2 + d*x/2)**3/(6291456*d*tan(c/2 + d*x/2)**
6 - 4718592*d*tan(c/2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d
) - 8940*tan(c/2 + d*x/2)/(6291456*d*tan(c/2 + d*x/2)**6 - 4718592*d*tan(c/
2 + d*x/2)**4 + 1179648*d*tan(c/2 + d*x/2)**2 - 98304*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.28

$$\int \frac{1}{(-3+5\cos(c+dx))^4}\,dx = -\frac{\frac{20\left(\frac{447\sin(dx+c)}{\cos(dx+c)+1}-\frac{1696\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2832\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{48\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{64\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-1}-837\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+1\right)+837\log\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{98304\,d}$$

```
[In] integrate(1/(-3+5*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/98304*(20*(447*sin(d*x + c)/(cos(d*x + c) + 1) - 1696*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 2832*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(12*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 48*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 64*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 1) - 837*log(2*sin(d*x + c)/(cos(d*x +
c) + 1) + 1) + 837*log(2*sin(d*x + c)/(cos(d*x + c) + 1) - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.70

$$\int \frac{1}{(-3+5\cos(c+dx))^4}\,dx = -\frac{\frac{20\left(2832\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-1696\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+447\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}-837\log\left(\left|2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)+837\log\left(\left|2\right.\right.}{98304\,d}$$

```
[In] integrate(1/(-3+5*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/98304*(20*(2832*tan(1/2*d*x + 1/2*c)^5 - 1696*tan(1/2*d*x + 1/2*c)^3 + 4
47*tan(1/2*d*x + 1/2*c))/(4*tan(1/2*d*x + 1/2*c)^2 - 1)^3 - 837*log(abs(2*t
an(1/2*d*x + 1/2*c) + 1)) + 837*log(abs(2*tan(1/2*d*x + 1/2*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-3+5\cos(c+dx))^4}\,dx = \frac{279\,\mathrm{atanh}\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{16384\,d} - \frac{\frac{295\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{32768}-\frac{265\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{49152}+\frac{745\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{524288}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{4}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16}-\frac{1}{64}\right)}$$

```
[In] int(1/(5*cos(c + d*x) - 3)^4,x)
```

```
[Out] (279*atanh(2*tan(c/2 + (d*x)/2)))/(16384*d) - ((745*tan(c/2 + (d*x)/2))/524
288 - (265*tan(c/2 + (d*x)/2)^3)/49152 + (295*tan(c/2 + (d*x)/2)^5)/32768)/
(d*((3*tan(c/2 + (d*x)/2)^2)/16 - (3*tan(c/2 + (d*x)/2)^4)/4 + tan(c/2 + (d
*x)/2)^6 - 1/64))
```

3.46 $\int \frac{1}{-3-5\cos(c+dx)}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \frac{1}{-3-5\cos(c+dx)}\,dx = \frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[Out] 1/4*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-1/4*ln(2*cos(1/2*d*x+1/2*
c)+sin(1/2*d*x+1/2*c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2738, 213}

$$\int \frac{1}{-3-5\cos(c+dx)}\,dx = \frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[In] Int[(-3 - 5*Cos[c + d*x])^(-1),x]
```

```
[Out] Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/(4*d) - Log[2*Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]]/(4*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{1}{-8+2x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$
$$= \frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{1}{-3-5\cos(c+dx)}\,dx = \frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[In] Integrate[(-3 - 5*Cos[c + d*x])^(-1),x]
[Out] Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/(4*d) - Log[2*Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]]/(4*d)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.51

method	result	size
parallelrisch	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{4d}$	33
derivativedivides	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{4}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{4}}{d}$	34
default	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{4}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{4}}{d}$	34
norman	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{4d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{4d}$	36
risch	$\frac{\ln\left(e^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5}\right)}{4d}-\frac{\ln\left(e^{i(dx+c)}+\frac{3}{5}+\frac{4i}{5}\right)}{4d}$	40

```
[In] int(1/(-3-5*cos(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/4*(ln(tan(1/2*d*x+1/2*c)-2)-ln(tan(1/2*d*x+1/2*c)+2))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{1}{-3-5\cos(c+dx)}\,dx = -\frac{\log\left(\frac{3}{2}\cos\left(dx+c\right)+2\sin\left(dx+c\right)+\frac{5}{2}\right)-\log\left(\frac{3}{2}\cos\left(dx+c\right)-2\sin\left(dx+c\right)+\frac{5}{2}\right)}{8\,d}$$

```
[In] integrate(1/(-3-5*cos(d*x+c)),x, algorithm="fricas")
[Out] -1/8*(log(3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2) - log(3/2*cos(d*x + c) -
 2*sin(d*x + c) + 5/2))/d
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{1}{-3-5\cos(c+dx)}\,dx = \begin{cases} \frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{4d}-\frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)}{4d} & \text{for}\ d\neq 0 \\ \frac{x}{-5\cos\left(c\right)-3} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(-3-5*cos(d*x+c)),x)
[Out] Piecewise((log(tan(c/2 + d*x/2) - 2)/(4*d) - log(tan(c/2 + d*x/2) + 2)/(4*d
), Ne(d, 0)), (x/(-5*cos(c) - 3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{1}{-3-5\cos(c+dx)}\,dx = -\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right)-\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{4\,d}$$

```
[In] integrate(1/(-3-5*cos(d*x+c)),x, algorithm="maxima")
[Out] -1/4*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - log(sin(d*x + c)/(cos(d*x
+ c) + 1) - 2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{1}{-3-5\cos(c+dx)}\,dx = -\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)-\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{4\,d}$$

```
[In] integrate(1/(-3-5*cos(d*x+c)),x, algorithm="giac")
[Out] -1/4*(log(abs(tan(1/2*d*x + 1/2*c) + 2)) - log(abs(tan(1/2*d*x + 1/2*c) - 2
)))/d
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.26

$$\int \frac{1}{-3-5\cos(c+dx)}\,dx = -\frac{\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{2\,d}$$

```
[In] int(-1/(5*cos(c + d*x) + 3),x)
[Out] -atanh(tan(c/2 + (d*x)/2)/2)/(2*d)
```

3.47 $\int \frac{1}{(-3-5\cos(c+dx))^2}\,dx$

Optimal result 321
Rubi [A] (verified) 321
Mathematica [A] (verified) 323
Maple [A] (verified) 323
Fricas [A] (verification not implemented) 324
Sympy [B] (verification not implemented) 324
Maxima [A] (verification not implemented) 325
Giac [A] (verification not implemented) 325
Mupad [B] (verification not implemented) 325

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{1}{(-3-5\cos(c+dx))^2}\,dx = \frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{5\sin(c+dx)}{16d(3+5\cos(c+dx))}$$

```
[Out] 3/64*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-3/64*ln(2*cos(1/2*d*x+1/
2*c)+sin(1/2*d*x+1/2*c))/d+5/16*sin(d*x+c)/d/(3+5*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2743, 12, 2738, 213}

$$\int \frac{1}{(-3-5\cos(c+dx))^2}\,dx = \frac{5\sin(c+dx)}{16d(5\cos(c+dx)+3)} + \frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{64d}$$

```
[In] Int[(-3 - 5*Cos[c + d*x])^(-2),x]
```

```
[Out] (3*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(64*d) - (3*Log[2*Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]])/(64*d) + (5*Sin[c + d*x])/(16*d*(3 + 5*Cos[c +
d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{5\sin(c+dx)}{16d(3+5\cos(c+dx))}+\frac{1}{16}\int\frac{3}{-3-5\cos(c+dx)}\,dx\\
&= \frac{5\sin(c+dx)}{16d(3+5\cos(c+dx))}+\frac{3}{16}\int\frac{1}{-3-5\cos(c+dx)}\,dx\\
&= \frac{5\sin(c+dx)}{16d(3+5\cos(c+dx))}+\frac{3\text{Subst}\left(\int\frac{1}{-8+2x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{8d}\\
&= \frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d}\\
&\quad-\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64d}+\frac{5\sin(c+dx)}{16d(3+5\cos(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.59

$$\int \frac{1}{(-3-5\cos(c+dx))^2}\,dx$$
$$= \frac{9\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+15\cos(c+dx)\left(\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\right.}{64d(3+5\,\mathrm{co}}$$

```
[In] Integrate[(-3 - 5*Cos[c + d*x])^(-2),x]
```

```
[Out] (9*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 15*Cos[c + d*x]*(Log[2*Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]) - 9*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 20*Sin[c + d*x])/(64*d
*(3 + 5*Cos[c + d*x]))
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{-\frac{5}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{64}-\frac{5}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{64}}{d}$
default	$\frac{-\frac{5}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{64}-\frac{5}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{64}}{d}$
norman	$-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-4\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{64d}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{64d}$
risch	$\frac{i\left(3\,\mathrm{e}^{i(dx+c)}+5\right)}{8d\left(5\,\mathrm{e}^{2i(dx+c)}+6\,\mathrm{e}^{i(dx+c)}+5\right)}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+\frac{3}{5}+\frac{4i}{5}\right)}{64d}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5}\right)}{64d}$
parallelrisch	$\frac{15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)\cos(dx+c)-15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)\cos(dx+c)+20\sin(dx+c)+9\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)-9\ln\left(\tan\right.}{64d(3+5\cos(dx+c))}$

```
[In] int(1/(-3-5*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-5/32/(tan(1/2*d*x+1/2*c)+2)-3/64*ln(tan(1/2*d*x+1/2*c)+2)-5/32/(tan(1
/2*d*x+1/2*c)-2)+3/64*ln(tan(1/2*d*x+1/2*c)-2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{1}{(-3-5\cos(c+dx))^2}\,dx = -\frac{3\,(5\,\cos\left(dx+c\right)+3)\log\left(\frac{3}{2}\,\cos\left(dx+c\right)+2\,\sin\left(dx+c\right)+\frac{5}{2}\right)-3\,(5\,\cos\left(dx+c\right)+3)\log\left(\frac{3}{2}\,\cos\left(dx+c\right.\right.}{128\,(5\,d\cos\left(dx+c\right)+3\,d)}$$

```
[In] integrate(1/(-3-5*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/128*(3*(5*cos(d*x + c) + 3)*log(3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2)
 - 3*(5*cos(d*x + c) + 3)*log(3/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) - 40
*sin(d*x + c))/(5*d*cos(d*x + c) + 3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(78) = 156$.

Time = 0.68 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.57

$$\int \frac{1}{(-3-5\cos(c+dx))^2}\,dx = \begin{cases} \frac{x}{(-3-5\cos\left(2\,\mathrm{atan}\left(2\right)\right))^2} \\ \frac{x}{(-5\cos\left(c\right)-3)^2} \\ \frac{3\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} - \frac{12\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{64d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} - \frac{3\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} + \frac{12\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)}{64d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} - \end{cases}$$

```
[In] integrate(1/(-3-5*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((x/(-3 - 5*cos(2*atan(2)))**2, Eq(c, -d*x - 2*atan(2)) | Eq(c, -d
*x + 2*atan(2))), (x/(-5*cos(c) - 3)**2, Eq(d, 0)), (3*log(tan(c/2 + d*x/2)
 - 2)*tan(c/2 + d*x/2)**2/(64*d*tan(c/2 + d*x/2)**2 - 256*d) - 12*log(tan(c
/2 + d*x/2) - 2)/(64*d*tan(c/2 + d*x/2)**2 - 256*d) - 3*log(tan(c/2 + d*x/2
) + 2)*tan(c/2 + d*x/2)**2/(64*d*tan(c/2 + d*x/2)**2 - 256*d) + 12*log(tan(
c/2 + d*x/2) + 2)/(64*d*tan(c/2 + d*x/2)**2 - 256*d) - 20*tan(c/2 + d*x/2)/
(64*d*tan(c/2 + d*x/2)**2 - 256*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int \frac{1}{(-3-5\cos(c+dx))^2}\,dx = -\frac{\frac{20\,\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-4\right)(\cos(dx+c)+1)} + 3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right) - 3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{64\,d}$$

```
[In] integrate(1/(-3-5*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/64*(20*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4)*(cos(d*x
+ c) + 1)) + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 3*log(sin(d*x + c
)/(cos(d*x + c) + 1) - 2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-3-5\cos(c+dx))^2}\,dx = -\frac{\frac{20\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4} + 3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right) - 3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{64\,d}$$

```
[In] integrate(1/(-3-5*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/64*(20*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 4) + 3*log(abs(tan
(1/2*d*x + 1/2*c) + 2)) - 3*log(abs(tan(1/2*d*x + 1/2*c) - 2)))/d
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int \frac{1}{(-3-5\cos(c+dx))^2}\,dx = -\frac{3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{32\,d} - \frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\right)}$$

```
[In] int(1/(5*cos(c + d*x) + 3)^2,x)
```

```
[Out] - (3*atanh(tan(c/2 + (d*x)/2)/2))/(32*d) - (5*tan(c/2 + (d*x)/2))/(16*d*(ta
n(c/2 + (d*x)/2)^2 - 4))
```

3.48 $\int \frac{1}{(-3-5\cos(c+dx))^3}\,dx$

Optimal result 326
Rubi [A] (verified) 326
Mathematica [A] (verified) 328
Maple [A] (verified) 329
Fricas [A] (verification not implemented) 329
Sympy [B] (verification not implemented) 330
Maxima [A] (verification not implemented) 330
Giac [A] (verification not implemented) 331
Mupad [B] (verification not implemented) 331

Optimal result

Integrand size = 12, antiderivative size = 115

$$\int \frac{1}{(-3-5\cos(c+dx))^3}\,dx = \frac{43\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{43\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{5\sin(c+dx)}{32d(3+5\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(3+5\cos(c+dx))}$$

```
[Out] 43/2048*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-43/2048*ln(2*cos(1/2*
d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d-5/32*sin(d*x+c)/d/(3+5*cos(d*x+c))^2+45/51
2*sin(d*x+c)/d/(3+5*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2743, 2833, 12, 2738, 213}

$$\int \frac{1}{(-3-5\cos(c+dx))^3}\,dx = \frac{45\sin(c+dx)}{512d(5\cos(c+dx)+3)} - \frac{5\sin(c+dx)}{32d(5\cos(c+dx)+3)^2} + \frac{43\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{43\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}$$

```
[In] Int[(-3 - 5*Cos[c + d*x])^(-3),x]
```

```
[Out] (43*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2048*d) - (43*Log[2*Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]])/(2048*d) - (5*Sin[c + d*x])/(32*d*(3 + 5*Co
s[c + d*x])^2) + (45*Sin[c + d*x])/(512*d*(3 + 5*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{5\sin(c+dx)}{32d(3+5\cos(c+dx))^2}+\frac{1}{32}\int\frac{6-5\cos(c+dx)}{(-3-5\cos(c+dx))^2}\,dx\\ &= -\frac{5\sin(c+dx)}{32d(3+5\cos(c+dx))^2}+\frac{45\sin(c+dx)}{512d(3+5\cos(c+dx))}+\frac{1}{512}\int\frac{43}{-3-5\cos(c+dx)}\,dx\end{aligned}$$

$$= -\frac{5\sin(c+dx)}{32d(3+5\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(3+5\cos(c+dx))} + \frac{43}{512}\int \frac{1}{-3-5\cos(c+dx)}\,dx$$

$$= -\frac{5\sin(c+dx)}{32d(3+5\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(3+5\cos(c+dx))} + \frac{43\text{Subst}\left(\int \frac{1}{-8+2x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{256d}$$

$$= \frac{43\log\left(2\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{43\log\left(2\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{5\sin(c+dx)}{32d(3+5\cos(c+dx))^2} + \frac{45\sin(c+dx)}{512d(3+5\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.89

$$\int \frac{1}{(-3-5\cos(c+dx))^3}\,dx = \frac{43\log\left(2\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{43\log\left(2\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{5}{512d\left(2\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{45\sin\left(\frac{1}{2}(c+dx)\right)}{2048d\left(2\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{5}{512d\left(2\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{45\sin\left(\frac{1}{2}(c+dx)\right)}{2048d\left(2\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

```
[In] Integrate[(-3 - 5*Cos[c + d*x])^(-3),x]
```

```
[Out] (43*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2048*d) - (43*Log[2*Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]])/(2048*d) - 5/(512*d*(2*Cos[(c + d*x)/2] - S
in[(c + d*x)/2])^2) + (45*Sin[(c + d*x)/2])/(2048*d*(2*Cos[(c + d*x)/2] - S
in[(c + d*x)/2])) + 5/(512*d*(2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (
45*Sin[(c + d*x)/2])/(2048*d*(2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

method	result
norman	$\frac{\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{128d}-\frac{85\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-4\right)^2}+\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{2048d}-\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{2048d}$
derivativedivides	$\frac{\frac{25}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}-\frac{85}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{2048}-\frac{25}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}-\frac{85}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{43\ln}{}}{d}$
default	$\frac{\frac{25}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}-\frac{85}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{43\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{2048}-\frac{25}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}-\frac{85}{1024\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{43\ln}{}}{d}$
risch	$\frac{i\left(215\,e^{3i(dx+c)}+387\,e^{2i(dx+c)}+325\,e^{i(dx+c)}+225\right)}{256d\left(5\,e^{2i(dx+c)}+6\,e^{i(dx+c)}+5\right)^2}-\frac{43\ln\left(e^{i(dx+c)}+\frac{3}{5}+\frac{4i}{5}\right)}{2048d}+\frac{43\ln\left(e^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5}\right)}{2048d}$
parallelrisch	$\frac{(2580\cos(dx+c)+1075\cos(2dx+2c)+1849)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)+(-2580\cos(dx+c)-1075\cos(2dx+2c)-1849)\ln\left(\tan\right.}{2048d(43+25\cos(2dx+2c)+60\cos(dx+c))}$

```
[In] int(1/(-3-5*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (35/128/d*tan(1/2*d*x+1/2*c)-85/512/d*tan(1/2*d*x+1/2*c)^3)/(tan(1/2*d*x+1/
2*c)^2-4)^2+43/2048/d*ln(tan(1/2*d*x+1/2*c)-2)-43/2048/d*ln(tan(1/2*d*x+1/2
*c)+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.12

$$\int \frac{1}{(-3-5\cos(c+dx))^3}\,dx = -\frac{43\left(25\cos\left(dx+c\right)^2+30\cos\left(dx+c\right)+9\right)\log\left(\frac{3}{2}\cos\left(dx+c\right)+2\sin\left(dx+c\right)+\frac{5}{2}\right)-43\left(25\cos\left(dx+\right.\right.}{4096\left(25\,d\cos\left(dx+c\right)\right.}$$

```
[In] integrate(1/(-3-5*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4096*(43*(25*cos(d*x + c)^2 + 30*cos(d*x + c) + 9)*log(3/2*cos(d*x + c)
+ 2*sin(d*x + c) + 5/2) - 43*(25*cos(d*x + c)^2 + 30*cos(d*x + c) + 9)*log(
3/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) - 40*(45*cos(d*x + c) + 11)*sin(d*
x + c))/(25*d*cos(d*x + c)^2 + 30*d*cos(d*x + c) + 9*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(102) = 204$.

Time = 1.17 (sec) , antiderivative size = 478, normalized size of antiderivative = 4.16

$$\int \frac{1}{(-3-5\cos(c+dx))^3}\,dx = \begin{cases} \frac{x}{(-3-5\cos(2\,\mathrm{atan}(2)))^3} \\ \frac{x}{(-5\cos(c)-3)^3} \\ \frac{43\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2048d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+32768d} - \frac{344\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2048d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+32768d} + \frac{688\log\left(\tan\left(\frac{c}{2}\right.\right.}{2048d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d} \end{cases}$$

```
[In] integrate(1/(-3-5*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((x/(-3 - 5*cos(2*atan(2)))**3, Eq(c, -d*x - 2*atan(2)) | Eq(c, -d
*x + 2*atan(2))), (x/(-5*cos(c) - 3)**3, Eq(d, 0)), (43*log(tan(c/2 + d*x/2
) - 2)*tan(c/2 + d*x/2)**4/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 +
d*x/2)**2 + 32768*d) - 344*log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**2/(2
048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) + 688*lo
g(tan(c/2 + d*x/2) - 2)/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x
/2)**2 + 32768*d) - 43*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**4/(2048*
d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) + 344*log(ta
n(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**2/(2048*d*tan(c/2 + d*x/2)**4 - 16384
*d*tan(c/2 + d*x/2)**2 + 32768*d) - 688*log(tan(c/2 + d*x/2) + 2)/(2048*d*t
an(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 32768*d) - 340*tan(c/2 +
 d*x/2)**3/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 + d*x/2)**2 + 3276
8*d) + 560*tan(c/2 + d*x/2)/(2048*d*tan(c/2 + d*x/2)**4 - 16384*d*tan(c/2 +
 d*x/2)**2 + 32768*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.17

$$\int \frac{1}{(-3-5\cos(c+dx))^3}\,dx = -\frac{\frac{20\left(\frac{28\sin(dx+c)}{\cos(dx+c)+1}-\frac{17\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-16}+43\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right)-43\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{2048\,d}$$

```
[In] integrate(1/(-3-5*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2048*(20*(28*sin(d*x + c)/(cos(d*x + c) + 1) - 17*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 16) + 43*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 43*l
og(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\int \frac{1}{(-3-5\cos(c+dx))^3}\,dx = -\frac{\frac{20\left(17\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-28\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\right)^2}+43\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)-43\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{2048\,d}$$

```
[In] integrate(1/(-3-5*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2048*(20*(17*tan(1/2*d*x + 1/2*c)^3 - 28*tan(1/2*d*x + 1/2*c))/(tan(1/2*
d*x + 1/2*c)^2 - 4)^2 + 43*log(abs(tan(1/2*d*x + 1/2*c) + 2)) - 43*log(abs(
tan(1/2*d*x + 1/2*c) - 2)))/d
```

Mupad [B] (verification not implemented)

Time = 13.84 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

$$\int \frac{1}{(-3-5\cos(c+dx))^3}\,dx = \frac{\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128}-\frac{85\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{512}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+16\right)} - \frac{43\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{1024\,d}$$

```
[In] int(-1/(5*cos(c + d*x) + 3)^3,x)
```

```
[Out] ((35*tan(c/2 + (d*x)/2))/128 - (85*tan(c/2 + (d*x)/2)^3)/512)/(d*(tan(c/2 +
 (d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 + 16)) - (43*atanh(tan(c/2 + (d*x)/2)/
2))/(1024*d)
```

3.49 $\int \frac{1}{(-3-5\cos(c+dx))^4}\,dx$

Optimal result . . . 332
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Optimal result

Integrand size = 12, antiderivative size = 140

$$\int \frac{1}{(-3-5\cos(c+dx))^4}\,dx = \frac{279\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} - \frac{279\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{5\sin(c+dx)}{48d(3+5\cos(c+dx))^3} - \frac{25\sin(c+dx)}{512d(3+5\cos(c+dx))^2} + \frac{995\sin(c+dx)}{24576d(3+5\cos(c+dx))}$$

```
[Out] 279/32768*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-279/32768*ln(2*cos(
1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+5/48*sin(d*x+c)/d/(3+5*cos(d*x+c))^3-2
5/512*sin(d*x+c)/d/(3+5*cos(d*x+c))^2+995/24576*sin(d*x+c)/d/(3+5*cos(d*x+c
))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used

$= \{2743, 2833, 12, 2738, 213\}$

$$\int \frac{1}{(-3-5\cos(c+dx))^4}\,dx = \frac{995\sin(c+dx)}{24576d(5\cos(c+dx)+3)} - \frac{25\sin(c+dx)}{512d(5\cos(c+dx)+3)^2} + \frac{5\sin(c+dx)}{48d(5\cos(c+dx)+3)^3} + \frac{279\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} - \frac{279\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}$$

```
[In] Int[(-3 - 5*Cos[c + d*x])^(-4),x]
```

```
[Out] (279*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(32768*d) - (279*Log[2*Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]])/(32768*d) + (5*Sin[c + d*x])/(48*d*(3 +
5*Cos[c + d*x])^3) - (25*Sin[c + d*x])/(512*d*(3 + 5*Cos[c + d*x])^2) + (99
5*Sin[c + d*x])/(24576*d*(3 + 5*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
```

```
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{5\sin(c+dx)}{48d(3+5\cos(c+dx))^3}+\frac{1}{48}\int\frac{9-10\cos(c+dx)}{(-3-5\cos(c+dx))^3}\,dx\\
&= \frac{5\sin(c+dx)}{48d(3+5\cos(c+dx))^3}-\frac{25\sin(c+dx)}{512d(3+5\cos(c+dx))^2}+\frac{\int\frac{154-75\cos(c+dx)}{(-3-5\cos(c+dx))^2}\,dx}{1536}\\
&= \frac{5\sin(c+dx)}{48d(3+5\cos(c+dx))^3}-\frac{25\sin(c+dx)}{512d(3+5\cos(c+dx))^2}\\
&\quad+\frac{995\sin(c+dx)}{24576d(3+5\cos(c+dx))}+\frac{\int\frac{837}{-3-5\cos(c+dx)}\,dx}{24576}\\
&= \frac{5\sin(c+dx)}{48d(3+5\cos(c+dx))^3}-\frac{25\sin(c+dx)}{512d(3+5\cos(c+dx))^2}\\
&\quad+\frac{995\sin(c+dx)}{24576d(3+5\cos(c+dx))}+\frac{279\int\frac{1}{-3-5\cos(c+dx)}\,dx}{8192}\\
&= \frac{5\sin(c+dx)}{48d(3+5\cos(c+dx))^3}-\frac{25\sin(c+dx)}{512d(3+5\cos(c+dx))^2}\\
&\quad+\frac{995\sin(c+dx)}{24576d(3+5\cos(c+dx))}+\frac{279\text{Subst}\left(\int\frac{1}{-8+2x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{4096d}\\
&= \frac{279\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}\\
&\quad-\frac{279\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}+\frac{5\sin(c+dx)}{48d(3+5\cos(c+dx))^3}\\
&\quad-\frac{25\sin(c+dx)}{512d(3+5\cos(c+dx))^2}+\frac{995\sin(c+dx)}{24576d(3+5\cos(c+dx))}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 296 vs. $2(140) = 280$.

Time = 0.01 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.11

$$\int \frac{1}{(-3-5\cos(c+dx))^4}\,dx = \frac{467046\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+104625\cos(3(c+dx))\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(\cdots\right.\right.}{\cdots}$$

```
[In] Integrate[(-3 - 5*Cos[c + d*x])^(-4),x]
```

```
[Out] (467046*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 104625*Cos[3*(c + d*x)
]*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 765855*Cos[c + d*x]*(Log[2*C
os[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]]) + 376650*Cos[2*(c + d*x)]*(Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 467046*Log[2*Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] - 104625*Cos[3*(c + d*x)]*Log[2*Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] + 226140*Sin[c + d*x] + 190800*Sin[2*(c + d*x)] + 99500*
Sin[3*(c + d*x)])/(393216*d*(3 + 5*Cos[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.71

method	result
norman	$\frac{-\frac{295\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512d}+\frac{265\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768d}-\frac{745\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8192d}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-4\right)^{3}}+\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{32768d}-\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{32768d}$
derivativedivides	$\frac{-\frac{125}{6144\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{3}}-\frac{175}{4096\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{2}}-\frac{745}{16384\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{32768}-\frac{125}{6144\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{3}}\cdots}{d}$
default	$\frac{-\frac{125}{6144\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{3}}-\frac{175}{4096\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{2}}-\frac{745}{16384\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{279\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{32768}-\frac{125}{6144\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{3}}\cdots}{d}$
risch	$\frac{i\left(20925\,e^{5i(dx+c)}+62775\,e^{4i(dx+c)}+111042\,e^{3i(dx+c)}+119310\,e^{2i(dx+c)}+68625\,e^{i(dx+c)}+24875\right)}{12288d\left(5\,e^{2i(dx+c)}+6\,e^{i(dx+c)}+5\right)^{3}}+\frac{279\ln\left(e^{i(dx+c)}+\cdots\right.}{32768d}$
parallelrisch	$\frac{(765855\cos(dx+c)+376650\cos(2dx+2c)+104625\cos(3dx+3c)+467046)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)+(-765855\cos(dx+c)-3\cdots}{98304d(558+125\cos(3dx\cdots}$

```
[In] int(1/(-3-5*cos(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-295/512/d*tan(1/2*d*x+1/2*c)+265/768/d*tan(1/2*d*x+1/2*c)^3-745/8192/d*ta
n(1/2*d*x+1/2*c)^5)/(tan(1/2*d*x+1/2*c)^2-4)^3+279/32768/d*ln(tan(1/2*d*x+1
/2*c)-2)-279/32768/d*ln(tan(1/2*d*x+1/2*c)+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.21

$$\int \frac{1}{(-3-5\cos(c+dx))^4}\,dx =$$
$$-\frac{837\left(125\cos\left(dx+c\right)^3+225\cos\left(dx+c\right)^2+135\cos\left(dx+c\right)+27\right)\log\left(\frac{3}{2}\cos\left(dx+c\right)+2\sin\left(dx+c\right)\right.}{}$$

```
[In] integrate(1/(-3-5*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/196608*(837*(125*cos(d*x + c)^3 + 225*cos(d*x + c)^2 + 135*cos(d*x + c)
+ 27)*log(3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2) - 837*(125*cos(d*x + c)^
3 + 225*cos(d*x + c)^2 + 135*cos(d*x + c) + 27)*log(3/2*cos(d*x + c) - 2*si
n(d*x + c) + 5/2) - 40*(4975*cos(d*x + c)^2 + 4770*cos(d*x + c) + 1583)*sin
(d*x + c))/(125*d*cos(d*x + c)^3 + 225*d*cos(d*x + c)^2 + 135*d*cos(d*x + c
) + 27*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. 2(126) = 252.

Time = 2.44 (sec) , antiderivative size = 816, normalized size of antiderivative = 5.83

$$\int \frac{1}{(-3-5\cos(c+dx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-3-5*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((x/(-3 - 5*cos(2*atan(2)))**4, Eq(c, -d*x - 2*atan(2)) | Eq(c, -d
*x + 2*atan(2))), (x/(-5*cos(c) - 3)**4, Eq(d, 0)), (837*log(tan(c/2 + d*x/
2) - 2)*tan(c/2 + d*x/2)**6/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/
2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 10044*log(tan(
c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**4/(98304*d*tan(c/2 + d*x/2)**6 - 117964
8*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) + 4017
6*log(tan(c/2 + d*x/2) - 2)*tan(c/2 + d*x/2)**2/(98304*d*tan(c/2 + d*x/2)**
6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456
*d) - 53568*log(tan(c/2 + d*x/2) - 2)/(98304*d*tan(c/2 + d*x/2)**6 - 117964
8*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 837*
log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**6/(98304*d*tan(c/2 + d*x/2)**6
- 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d
) + 10044*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**4/(98304*d*tan(c/2 +
d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 -
 6291456*d) - 40176*log(tan(c/2 + d*x/2) + 2)*tan(c/2 + d*x/2)**2/(98304*d*
```

```
tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d
*x/2)**2 - 6291456*d) + 53568*log(tan(c/2 + d*x/2) + 2)/(98304*d*tan(c/2 +
d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 -
 6291456*d) - 8940*tan(c/2 + d*x/2)**5/(98304*d*tan(c/2 + d*x/2)**6 - 11796
48*d*tan(c/2 + d*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) + 339
20*tan(c/2 + d*x/2)**3/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d
*x/2)**4 + 4718592*d*tan(c/2 + d*x/2)**2 - 6291456*d) - 56640*tan(c/2 + d*x
/2)/(98304*d*tan(c/2 + d*x/2)**6 - 1179648*d*tan(c/2 + d*x/2)**4 + 4718592*
d*tan(c/2 + d*x/2)**2 - 6291456*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.24

$$\int \frac{1}{(-3-5\cos(c+dx))^4}\,dx = -\frac{\frac{20\left(\frac{2832\sin(dx+c)}{\cos(dx+c)+1}-\frac{1696\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{447\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{12\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-64}+837\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right)-837\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{98304\,d}$$

```
[In] integrate(1/(-3-5*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/98304*(20*(2832*sin(d*x + c)/(cos(d*x + c) + 1) - 1696*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 + 447*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 - 64) + 837*log(sin(d*x + c)/(cos(d*x + c) +
 1) + 2) - 837*log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

$$\int \frac{1}{(-3-5\cos(c+dx))^4}\,dx = -\frac{\frac{20\left(447\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1696\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2832\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\right)^3}+837\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)-837\log\left(\left|\mathrm{tar}\right.\right.}{98304\,d}$$

```
[In] integrate(1/(-3-5*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/98304*(20*(447*tan(1/2*d*x + 1/2*c)^5 - 1696*tan(1/2*d*x + 1/2*c)^3 + 28
32*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 4)^3 + 837*log(abs(tan(1
/2*d*x + 1/2*c) + 2)) - 837*log(abs(tan(1/2*d*x + 1/2*c) - 2)))/d
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-3-5\cos(c+dx))^4}\,dx = -\frac{279\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{16384\,d} - \frac{\frac{745\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8192}-\frac{265\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{768}+\frac{295\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{512}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-64\right)}$$

```
[In] int(1/(5*cos(c + d*x) + 3)^4,x)
```

```
[Out] - (279*atanh(tan(c/2 + (d*x)/2)/2))/(16384*d) - ((295*tan(c/2 + (d*x)/2))/5
12 - (265*tan(c/2 + (d*x)/2)^3)/768 + (745*tan(c/2 + (d*x)/2)^5)/8192)/(d*(
48*tan(c/2 + (d*x)/2)^2 - 12*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 -
64))
```

3.50 $\int (a+b\cos(c+dx))^{5/2}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 197

$$\int (a+b\cos(c+dx))^{5/2}\,dx=\frac{2(23a^2+9b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{16a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{16ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

```
[Out] 2/5*b*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+16/15*a*b*sin(d*x+c)*(a+b*cos(d*x
+c))^(1/2)/d+2/15*(23*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c)
)^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-16/15*a*(a^2-b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+
b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

$= \{2735, 2832, 2831, 2742, 2740, 2734, 2732\}$

$$\int (a+b\cos(c+dx))^{5/2}\,dx = -\frac{16a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}} + \frac{2(23a^2+9b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d} + \frac{16ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{15d}$$

```
[In] Int[(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(23*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (16*a*(a^2 - b^2)*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*d
*Sqrt[a + b*Cos[c + d*x]]) + (16*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(15*d) + (2*b*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} \\
&\quad + \frac{2}{5}\int \sqrt{a+b\cos(c+dx)}\left(\frac{1}{2}\left(5a^2+3b^2\right)+4ab\cos(c+dx)\right)\,dx \\
&= \frac{16ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} \\
&\quad + \frac{4}{15}\int \frac{\frac{1}{4}a\left(15a^2+17b^2\right)+\frac{1}{4}b\left(23a^2+9b^2\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx \\
&= \frac{16ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} \\
&\quad - \frac{1}{15}\left(8a\left(a^2-b^2\right)\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx+\frac{1}{15}\left(23a^2\right. \\
&\qquad \left.+9b^2\right)\int \sqrt{a+b\cos(c+dx)}\,dx
\end{aligned}$$

$$= \frac{16ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{\left((23a^2+9b^2)\sqrt{a+b\cos(c+dx)}\right)\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{15\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{\left(8a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{15\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2(23a^2+9b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{16a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}} + \frac{16ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.90

$$\int(a+b\cos(c+dx))^{5/2}\,dx = \frac{2(23a^3+23a^2b+9ab^2+9b^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-16a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{Elli}}{15d\sqrt{a+b\text{co}}}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 16*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(22*a^2 + 3*b^2
 + 28*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(15*d*Sqrt[a
 + b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(235) = 470$.

Time = 6.61 (sec) , antiderivative size = 662, normalized size of antiderivative = 3.36

method	result
default	$-\frac{2\sqrt{\left(2b\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(24\left(\cos^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^{3}+56\left(\cos^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^{2}-48\left(\cos^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^{3}+22\left(\cos^{3}\left(\right.\right.\right.}{}$

```
[In] int((a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1
/2*d*x+1/2*c)^7*b^3+56*cos(1/2*d*x+1/2*c)^5*a*b^2-48*cos(1/2*d*x+1/2*c)^5*b
^3+22*cos(1/2*d*x+1/2*c)^3*a^2*b-84*cos(1/2*d*x+1/2*c)^3*a*b^2+30*cos(1/2*d
*x+1/2*c)^3*b^3-8*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)
^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+8*a*b
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+23*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^3-23*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/
2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a^2*b+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-22*cos(1/2*d*x+1/2*c)*a^2*b
+28*cos(1/2*d*x+1/2*c)*a*b^2-6*cos(1/2*d*x+1/2*c)*b^3)/(-2*sin(1/2*d*x+1/2*
c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d
*x+1/2*c)^2+a+b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.22

$$\int (a + b\cos(c + dx))^{5/2}\,dx = \frac{\sqrt{2}(i\,a^3 - 33i\,ab^2)\sqrt{b}\,\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2}, -\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3}, \frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2\,a}{3\,b}\right.}{}$$

```
[In] integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/45*(sqrt(2)*(I*a^3 - 33*I*a*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 -
 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin
```

```
(d*x + c) + 2*a)/b) + sqrt(2)*(-I*a^3 + 33*I*a*b^2)*sqrt(b)*weierstrassPInv
erse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x
 + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(-23*I*a^2*b - 9*I*b^3)*sq
rt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
 weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(23*I*a^2
*b + 9*I*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*(
3*b^3*cos(d*x + c) + 11*a*b^2)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b*d)
```

Sympy [F]

$$\int (a+b\cos(c+dx))^{5/2}\,dx=\int (a+b\cos{(c+dx)})^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**(5/2),x)
[Out] Integral((a + b*cos(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}\,dx=\int (b\cos{(dx+c)}+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}\,dx=\int (b\cos{(dx+c)}+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\,dx = \int (a+b\,\cos(c+d\,x))^{5/2}\,dx$$

```
[In] int((a + b*cos(c + d*x))^(5/2),x)
[Out] int((a + b*cos(c + d*x))^(5/2), x)
```

3.51 $\int (a+b\cos(c+dx))^{3/2}\,dx$

Optimal result 346
Rubi [A] (verified) 346
Mathematica [A] (verified) 348
Maple [B] (verified) 349
Fricas [C] (verification not implemented) 349
Sympy [F] 350
Maxima [F] 350
Giac [F] 350
Mupad [F(-1)] 350

Optimal result

Integrand size = 14, antiderivative size = 157

$$\int (a+b\cos(c+dx))^{3/2}\,dx=\frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}$$

```
[Out] 2/3*b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+8/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*
(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(a^2-b^2)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/
2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2735, 2831, 2742, 2740, 2734, 2732}

$$\int (a+b\cos(c+dx))^{3/2}\,dx=-\frac{2(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[In] Int[(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*Cos[c +
d*x]]) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}(3a^2+b^2)+2ab\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}(4a)\int \sqrt{a+b\cos(c+dx)}\,dx + \frac{1}{3}(-a^2+b^2)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{\left(4a\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{3\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\left((-a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{3\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}} + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int (a+b\cos(c+dx))^{3/2}\,dx = \frac{8a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right) - 2(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b
)/(a + b)] - 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
+ d*x)/2, (2*b)/(a + b)] + 2*b*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt
[a + b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(199) = 398$.

Time = 4.09 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.87

method	result
default	$-\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+2\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-6\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2-a^2\sqrt{\frac{1}{2}-\frac{\cos(dx+}{2}}\right.}{}$

```
[In] int((a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2
*d*x+1/2*c)^5*b^2+2*cos(1/2*d*x+1/2*c)^3*a*b-6*cos(1/2*d*x+1/2*c)^3*b^2-a^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+
1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^2-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*cos(1/2*d*x+1/
2*c)*a*b+2*cos(1/2*d*x+1/2*c)*b^2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/
2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.54

$$\int (a+b\cos(c+dx))^{3/2}\,dx = \frac{12i\sqrt{2}ab^{\frac{3}{2}}\,\text{weierstrassZeta}\left(\frac{4\left(4a^2-3b^2\right)}{3b^2}, -\frac{8\left(8a^3-9ab^2\right)}{27b^3}, \text{weierstrassPInverse}\left(\frac{4\left(4a^2-3b^2\right)}{3b^2}, -\frac{8\left(8a^3-\right.}{27b}\right.\right.}{}$$

```
[In] integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*(12*I*sqrt(2)*a*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*
(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(
8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b))
 - 12*I*sqrt(2)*a*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8
*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*
a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) +
 6*sqrt(b*cos(d*x + c) + a)*b^2*sin(d*x + c) + sqrt(2)*(-I*a^2 - 3*I*b^2)*s
```

```
qrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)
/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(I*a^2
 + 3*I*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a
^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b))/(b
*d)
```

Sympy [F]

$$\int (a+b\cos(c+dx))^{3/2}\,dx=\int \left(a+b\cos\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**(3/2),x)
[Out] Integral((a + b*cos(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\,dx=\int \left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

```
[In] int((a + b*cos(c + d*x))^(3/2),x)
[Out] int((a + b*cos(c + d*x))^(3/2), x)
```

3.52 $\int \sqrt{a+b\cos(c+dx)}\,dx$

Optimal result 351
Rubi [A] (verified) 351
Mathematica [A] (verified) 352
Maple [B] (verified) 352
Fricas [C] (verification not implemented) 353
Sympy [F] 353
Maxima [F] 354
Giac [F] 354
Mupad [F(-1)] 354

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \sqrt{a+b\cos(c+dx)}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+
b))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2734, 2732}

$$\int \sqrt{a+b\cos(c+dx)}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[In] Int[Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[
(a + b*Cos[c + d*x])/(a + b)])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\ &= \frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int\sqrt{a+b\cos(c+dx)}\,dx=\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[
(a + b*Cos[c + d*x])/(a + b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(82) = 164.

Time = 2.92 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.98

method	result	size
default	$-\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}{a-b}}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)(a-b)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}\,d}$	170
risch	Expression too large to display	1046

```
[In] int((a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos
```

```
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(a-b)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a
+b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 355, normalized size of antiderivative = 6.23

$$\int \sqrt{a+b\cos(c+dx)}\,dx = \frac{-i\sqrt{2}a\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2\,a}{3\,b}\right)+i\sqrt{2}a\sqrt{b}\text{weierstras}}{}$$

```
[In] integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/
b) + I*sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b
) + 3*I*sqrt(2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a
^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^
3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3
*I*sqrt(2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 -
9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9
*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/(b*d)
```

Sympy [F]

$$\int \sqrt{a+b\cos(c+dx)}\,dx = \int \sqrt{a+b\cos{(c+dx)}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}\,dx = \int \sqrt{b\cos\left(dx+c\right)+a}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}\,dx = \int \sqrt{b\cos\left(dx+c\right)+a}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}\,dx = \int \sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

```
[In] int((a + b*cos(c + d*x))^(1/2),x)
[Out] int((a + b*cos(c + d*x))^(1/2), x)
```

3.53 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result 355
Rubi [A] (verified) 355
Mathematica [A] (verified) 356
Maple [C] (verified) 356
Fricas [C] (verification not implemented) 357
Sympy [F] 357
Maxima [F] 357
Giac [F] 358
Mupad [B] (verification not implemented) 358

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx=\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+
c))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2742, 2740}

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx=\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[In] Int[1/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
)/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int\frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[In] Integrate[1/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
)/(d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.82 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.37

method	result	size
default	$\frac{2\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}{a+b}}\,\text{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\,\middle|\,\frac{\sqrt{2}\,\sqrt{b}}{\sqrt{a+b}}\right)}{d\sqrt{2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}}$	78

```
[In] int(1/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^(1/2)*(-(2*b*sin(1/2*d*x+1/2*c)^2-a-b)/(
a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)/(a+b)^(1/2)*b^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.56

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2}, -\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3}, \frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2\,a}{3\,b}\right) + i\sqrt{2}\sqrt{b}\text{weierstrassP}}{bd}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a
^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + I
*sqrt(2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b))/(b*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(b*cos(d*x + c) + a), x)

Mupad [B] (verification not implemented)

Time = 13.93 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\sqrt{\frac{a+b\,\cos(c+d\,x)}{a+b}}}{d\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}$$

[In] int(1/(a + b*cos(c + d*x))^(1/2),x)

[Out] (2*ellipticF(c/2 + (d*x)/2, (2*b)/(a + b))*((a + b*cos(c + d*x))/(a + b))^(1/2))/(d*(a + b*cos(c + d*x))^(1/2))

3.54 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result 359
Rubi [A] (verified) 359
Mathematica [A] (verified) 361
Maple [A] (verified) 361
Fricas [C] (verification not implemented) 361
Sympy [F] 362
Maxima [F] 362
Giac [F] 362
Mupad [F(-1)] 363

Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -2*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1
/2))*(a+b*cos(d*x+c))^(1/2)/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2743, 21, 2734, 2732}

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\,(a^2-b^2)\,\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2b\sin(c+dx)}{d\,(a^2-b^2)\,\sqrt{a+b\cos(c+dx)}}$$

```
[In] Int[(a + b*Cos[c + d*x])^(-3/2),x]
```

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/((a^2 -
b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*b*Sin[c + d*x])/((a^2 - b^2
)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}-\frac{2\int\frac{-\frac{a}{2}-\frac{1}{2}b\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2}\\
&= -\frac{2b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}+\frac{\int\sqrt{a+b\cos(c+dx)}\,dx}{a^2-b^2}\\
&= -\frac{2b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}+\frac{\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\
&= \frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2b\sin(c+dx)}{(a-b)(a+b)d\sqrt{a+b\cos(c+dx)}}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(-3/2),x]
```

```
[Out] (2*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/
(a + b)] - 2*b*Sin[c + d*x])/((a - b)*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.05

method	result
default	$-\frac{2\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,a-E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\right.\right.}{(a-b)(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}\,d}$

```
[In] int(1/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*b)/(a-b)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)
^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.55

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{6\sqrt{b\cos(dx+c)+a}b^2\sin(dx+c)+\left(i\sqrt{2}ab\cos(dx+c)+i\sqrt{2}a^2\right)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},\right.}{}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*sqrt(b*cos(d*x + c) + a)*b^2*sin(d*x + c) + (I*sqrt(2)*a*b*cos(d*x
+ c) + I*sqrt(2)*a^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
-8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2
*a)/b) + (-I*sqrt(2)*a*b*cos(d*x + c) - I*sqrt(2)*a^2)*sqrt(b)*weierstrassP
Inverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(
d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*(I*sqrt(2)*b^2*cos(d*x + c) + I
*sqrt(2)*a*b)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(
-I*sqrt(2)*b^2*cos(d*x + c) - I*sqrt(2)*a*b)*sqrt(b)*weierstrassZeta(4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*
b*sin(d*x + c) + 2*a)/b)))/((a^2*b^2 - b^4)*d*cos(d*x + c) + (a^3*b - a*b^3
)*d)
```

Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(a+b\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))**(3/2),x)
[Out] Integral((a + b*cos(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(1/(a + b*cos(c + d*x))^(3/2),x)
[Out] int(1/(a + b*cos(c + d*x))^(3/2), x)
```

3.55 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 221

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right) d\sqrt{a+b\cos(c+dx)}} - \frac{2b\sin(c+dx)}{3\left(a^2-b^2\right) d(a+b\cos(c+dx))^{3/2}} - \frac{8ab\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -2/3*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-8/3*a*b*sin(d*x+c)/(a^
2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+8/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos
(d*x+c))^(1/2)/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(
b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))
^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

= {2743, 2833, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{8ab\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}} - \frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}} - \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}} + \frac{8a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[In] Int[(a + b*Cos[c + d*x])^(-5/2),x]
```

```
[Out] (8*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*(a^
2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*(a^2 - b^2)*d*Sqrt[
a + b*Cos[c + d*x]]) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d
*x])^(3/2)) - (8*a*b*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*
x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}-\frac{2\int\frac{-\frac{3a}{2}+\frac{1}{2}b\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}\\
&= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}\\
&\quad -\frac{8ab\sin(c+dx)}{3\left(a^2-b^2\right)^2d\sqrt{a+b\cos(c+dx)}}+\frac{4\int\frac{\frac{1}{4}\left(3a^2+b^2\right)+ab\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{3\left(a^2-b^2\right)^2}\\
&= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}-\frac{8ab\sin(c+dx)}{3\left(a^2-b^2\right)^2d\sqrt{a+b\cos(c+dx)}}\\
&\quad +\frac{(4a)\int\sqrt{a+b\cos(c+dx)}\,dx}{3\left(a^2-b^2\right)^2}-\frac{\int\frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{3\left(a^2-b^2\right)}
\end{aligned}
$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8ab\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$
$$+ \frac{\left(4a\sqrt{a+b\cos(c+dx)}\right)\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{3\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$- \frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{3\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}$$
$$- \frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8ab\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71

$$\int\frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{8a(a+b)^2\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right) - 2(a-b)(a+b)^2\left(\frac{a+b\cos(c+dx)}{a+b}\right. \cdots}{3(a-b)^2(a+b)^2d(\cdots}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(-5/2),x]
[Out] (8*a*(a + b)^2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*EllipticE[(c + d*x)/2,
(2*b)/(a + b)] - 2*(a - b)*(a + b)^2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*E
llipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*(-5*a^2 + b^2 - 4*a*b*Cos[c + d*
x])*Sin[c + d*x])/(3*(a - b)^2*(a + b)^2*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.21

method	result
default	$-\frac{\sqrt{-\left(-2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{3b(a-b)(a+b)\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{a-b}{2b}\right)^2}+\frac{16\left(\sin^2\left(\cdots\right.\right.}{3(a-b)^2(a+b)^2\sqrt{-\left(-2b\left(\cdots\right.\right.}}\right. \cdots}{\cdots}$

```
[In] int(1/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1/3/b/(a-b)
/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+16/3*sin(1/2*d*x+1/2*c)^2*
b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)+2*(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))-8/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/
(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 692, normalized size of antiderivative = 3.13

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{6\left(4\,ab^3\cos\left(dx+c\right)+5\,a^2b^2-b^4\right)\sqrt{b\cos\left(dx+c\right)+a}\sin\left(dx+c\right)-\left(\sqrt{2}(-i\,a^2b^2-3i\,b^4)\cos\left(dx+c\right)^2-2\right.}{}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/9*(6*(4*a*b^3*cos(d*x + c) + 5*a^2*b^2 - b^4)*sqrt(b*cos(d*x + c) + a)*s
in(d*x + c) - (sqrt(2)*(-I*a^2*b^2 - 3*I*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(I
*a^3*b + 3*I*a*b^3)*cos(d*x + c) + sqrt(2)*(-I*a^4 - 3*I*a^2*b^2))*sqrt(b)*
weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1
/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) - (sqrt(2)*(I*a^2*b^2 +
 3*I*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(-I*a^3*b - 3*I*a*b^3)*cos(d*x + c) +
sqrt(2)*(I*a^4 + 3*I*a^2*b^2))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b
^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x
 + c) + 2*a)/b) + 12*(-I*sqrt(2)*a*b^3*cos(d*x + c)^2 - 2*I*sqrt(2)*a^2*b^2
*cos(d*x + c) - I*sqrt(2)*a^3*b)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2
)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)
/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x +
c) + 2*a)/b)) + 12*(I*sqrt(2)*a*b^3*cos(d*x + c)^2 + 2*I*sqrt(2)*a^2*b^2*co
s(d*x + c) + I*sqrt(2)*a^3*b)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b
^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c)
+ 2*a)/b)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*cos(d*x + c)^2 + 2*(a^5*b^2 - 2*
a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)
```

Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\cos{(c+dx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))**(5/2),x)
[Out] Integral((a + b*cos(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int(1/(a + b*cos(c + d*x))^(5/2),x)
[Out] int(1/(a + b*cos(c + d*x))^(5/2), x)
```

3.56 $\int (a+b\cos(c+dx))^{4/3}\,dx$

Optimal result 370
Rubi [A] (verified) 370
Mathematica [B] (verified) 372
Maple [F] 372
Fricas [F] 372
Sympy [F] 373
Maxima [F] 373
Giac [F] 373
Mupad [F(-1)] 373

Optimal result

Integrand size = 14, antiderivative size = 108

$$\int (a+b\cos(c+dx))^{4/3}\,dx=\frac{\sqrt{2}(a+b)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+ \cdots}{d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (a+b)*AppellF1(1/2,-4/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*
(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/
(1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2744, 144, 143}

$$\int (a+b\cos(c+dx))^{4/3}\,dx=\frac{\sqrt{2}(a+b)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx \cdots}{a+b}\right.}{d\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[In] Int[(a + b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1
- Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*Sqrt[
1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{\sin(c+dx)\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}$$

$$= \frac{\left((-a-b)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{4/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}}$$

$$= \frac{\sqrt{2}(a+b)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx}{d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 246 vs. $2(108) = 216$.

Time = 2.07 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.28

$$\int (a+b\cos(c+dx))^{4/3}\,dx =$$
$$-\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(4(-a^2+b^2)\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\right.}{}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-a^2 + b^2)*AppellF1[1/3, 1
/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*S
qrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a -
b))] + 5*a*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a +
b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b
*(1 + Cos[c + d*x]))/(a - b))]*(a + b*Cos[c + d*x]) - 4*b^2*Sin[c + d*x]^2)
)/(16*b*d)
```

Maple [F]

$$\int (a+\cos(dx+c)\,b)^{\frac{4}{3}}\,dx$$

```
[In] int((a+cos(d*x+c)*b)^(4/3),x)
```

```
[Out] int((a+cos(d*x+c)*b)^(4/3),x)
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{4/3}\,dx = \int (b\cos(dx+c)+a)^{\frac{4}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(4/3), x)
```

Sympy [F]

$$\int (a+b\cos(c+dx))^{4/3}\,dx=\int (a+b\cos(c+dx))^{\frac{4}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**(4/3),x)
[Out] Integral((a + b*cos(c + d*x))**(4/3), x)
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{4/3}\,dx=\int (b\cos(dx+c)+a)^{\frac{4}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(4/3), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{4/3}\,dx=\int (b\cos(dx+c)+a)^{\frac{4}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{4/3}\,dx=\int (a+b\,\cos(c+d\,x))^{4/3}\,dx$$

```
[In] int((a + b*cos(c + d*x))^(4/3),x)
[Out] int((a + b*cos(c + d*x))^(4/3), x)
```

3.57 $\int (a+b\cos(c+dx))^{2/3}\,dx$

Optimal result 374
Rubi [A] (verified) 374
Mathematica [A] (verified) 376
Maple [F] 376
Fricas [F] 376
Sympy [F] 376
Maxima [F] 377
Giac [F] 377
Mupad [F(-1)] 377

Optimal result

Integrand size = 14, antiderivative size = 105

$$\int (a+b\cos(c+dx))^{2/3}\,dx=\frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] AppellF1(1/2,-2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*c
os(d*x+c))^(2/3)*sin(d*x+c)*2^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(2/3)/(1+cos
(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2744, 144, 143}

$$\int (a+b\cos(c+dx))^{2/3}\,dx=\frac{\sqrt{2}\sin(c+dx)(a+b\cos(c+dx))^{2/3}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{d\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

```
[In] Int[(a + b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c
+ d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*Sqrt[1 + Cos[
c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}} \\
&= -\frac{\left((a+b\cos(c+dx))^{2/3}\sin(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}} \\
&= \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12

$$\int (a+b\cos(c+dx))^{2/3}\,dx = -\frac{3\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}(a+b\cos(c+dx))^{5/3}\csc(c+dx)}{5bd}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c
 + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos
[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])^(5/3)*Csc[c + d*x])/(5*b*d)
```

Maple [F]

$$\int \left(a+\cos\left(dx+c\right)b\right)^{\frac{2}{3}}dx$$

```
[In] int((a+cos(d*x+c)*b)^(2/3),x)
```

```
[Out] int((a+cos(d*x+c)*b)^(2/3),x)
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{2/3}\,dx = \int \left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}dx$$

```
[In] integrate((a+b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F]

$$\int (a+b\cos(c+dx))^{2/3}\,dx = \int \left(a+b\cos\left(c+dx\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{2/3}\,dx = \int (b\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(2/3), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{2/3}\,dx = \int (b\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{2/3}\,dx = \int (a+b\,\cos(c+d\,x))^{2/3}\,dx$$

```
[In] int((a + b*cos(c + d*x))^(2/3),x)
[Out] int((a + b*cos(c + d*x))^(2/3), x)
```

3.58 $\int \sqrt[3]{a+b\cos(c+dx)}\,dx$

Optimal result 378
Rubi [A] (verified) 378
Mathematica [A] (verified) 380
Maple [F] 380
Fricas [F] 380
Sympy [F] 380
Maxima [F] 381
Giac [F] 381
Mupad [F(-1)] 381

Optimal result

Integrand size = 14, antiderivative size = 105

$$\int \sqrt[3]{a+b\cos(c+dx)}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] AppellF1(1/2,-1/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*c
os(d*x+c))^(1/3)*sin(d*x+c)*2^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos
(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2744, 144, 143}

$$\int \sqrt[3]{a+b\cos(c+dx)}\,dx = \frac{\sqrt{2}\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{d\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[In] Int[(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c
+ d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*Sqrt[1 + Cos[
c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{\sin(c+dx)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}$$

$$= -\frac{\left(\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{-\frac{a}{-a-b}-\frac{bx}{-a-b}}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}}$$

$$= \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{3}{2}, \frac{1}{2}(1-\cos(c+dx)), \frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12

$$\int \sqrt[3]{a+b\cos(c+dx)}\,dx = -\frac{3\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}(a+b\cos(c+dx))^{4/3}\,\text{cs}}{4bd}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c
 + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos
[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])^(4/3)*Csc[c + d*x])/(4*b*d)
```

Maple [F]

$$\int (a+\cos(dx+c)\,b)^{\frac{1}{3}}\,dx$$

```
[In] int((a+cos(d*x+c)*b)^(1/3),x)
```

```
[Out] int((a+cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\,dx = \int (b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\,dx = \int \sqrt[3]{a+b\cos(c+dx)}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\,dx = \int (b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\,dx = \int (b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+b\cos(c+dx)}\,dx = \int (a+b\,\cos(c+d\,x))^{1/3}\,dx$$

```
[In] int((a + b*cos(c + d*x))^(1/3),x)
[Out] int((a + b*cos(c + d*x))^(1/3), x)
```

3.59 $\int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$

Optimal result . . . 382
Rubi [A] (verified) . . . 382
Mathematica [A] (verified) . . . 384
Maple [F] . . . 384
Fricas [F] . . . 384
Sympy [F] . . . 384
Maxima [F] . . . 385
Giac [F] . . . 385
Mupad [F(-1)] . . . 385

Optimal result

Integrand size = 14, antiderivative size = 105

$$\int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

```
[Out] AppellF1(1/2,1/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*c
os(d*x+c))/(a+b))^(1/3)*sin(d*x+c)*2^(1/2)/d/(a+b*cos(d*x+c))^(1/3)/(1+cos(
d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2744, 144, 143}

$$\int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \frac{\sqrt{2}\sin(c+dx)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{d\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}}$$

```
[In] Int[(a + b*Cos[c + d*x])^(-1/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c +
 d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/(d*Sqrt
[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{\sin(c+dx)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{a+bx}}\,dx, x, \cos(c+dx)\right)}{d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}$$

$$= -\frac{\left(\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}\sin(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{-\frac{a}{-a-b}-\frac{bx}{-a-b}}}\,dx, x, \cos(c+dx)\right)}{d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

$$= \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = -\frac{3\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}(a+b\cos(c+dx))^{2/3}\,\text{cs}}{2bd}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(-1/3),x]
```

```
[Out] (-3*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c
 + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos
[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x])/(2*b*d)
```

Maple [F]

$$\int \frac{1}{(a+\cos(dx+c)\,b)^{\frac{1}{3}}}dx$$

```
[In] int(1/(a+cos(d*x+c)*b)^(1/3),x)
```

```
[Out] int(1/(a+cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(-1/3), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{1}{(a+b\,\cos(c+d\,x))^{1/3}}\,dx$$

```
[In] int(1/(a + b*cos(c + d*x))^(1/3),x)
[Out] int(1/(a + b*cos(c + d*x))^(1/3), x)
```

3.60 $\int \frac{1}{(a+b\cos(c+dx))^{2/3}}\,dx$

Optimal result 386
Rubi [A] (verified) 386
Mathematica [A] (verified) 388
Maple [F] 388
Fricas [F] 388
Sympy [F] 388
Maxima [F] 389
Giac [F] 389
Mupad [F(-1)] 389

Optimal result

Integrand size = 14, antiderivative size = 105

$$\int \frac{1}{(a+b\cos(c+dx))^{2/3}}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\sin}{d\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}}$$

```
[Out] AppellF1(1/2,2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*c
os(d*x+c))/(a+b))^(2/3)*sin(d*x+c)*2^(1/2)/d/(a+b*cos(d*x+c))^(2/3)/(1+cos(
d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2744, 144, 143}

$$\int \frac{1}{(a+b\cos(c+dx))^{2/3}}\,dx = \frac{\sqrt{2}\sin(c+dx)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos}{}\right.}{d\sqrt{\cos(c+dx)+1}(a+b\cos(c+dx))^{2/3}}$$

```
[In] Int[(a + b*Cos[c + d*x])^(-2/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c +
 d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(d*Sqrt
[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
```

```
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{\sin(c+dx)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}(a+bx)^{2/3}}\,dx, x, \cos(c+dx)\right)}{d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}$$

$$= -\frac{\left(\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}\sin(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}\,dx, x, \cos(c+dx)\right)}{d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}}$$

$$= \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+b\cos(c+dx))^{2/3}}\,dx = -\frac{3\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\sqrt[3]{a+b\cos(c+dx)}\csc(c+}{bd}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(-2/3),x]
```

```
[Out] (-3*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c
 + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos
[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x])/(b*d)
```

Maple [F]

$$\int \frac{1}{(a+\cos(dx+c)\,b)^{\frac{2}{3}}}dx$$

```
[In] int(1/(a+cos(d*x+c)*b)^(2/3),x)
```

```
[Out] int(1/(a+cos(d*x+c)*b)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(-2/3), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{2/3}}\,dx = \int \frac{1}{(a+b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(a + b\cos(c + dx))^{2/3}}\,dx = \int \frac{1}{\left(b\cos\left(dx + c\right) + a\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(a + b\cos(c + dx))^{2/3}}\,dx = \int \frac{1}{\left(b\cos\left(dx + c\right) + a\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\cos(c + dx))^{2/3}}\,dx = \int \frac{1}{\left(a + b\,\cos\left(c + d\,x\right)\right)^{2/3}}\,dx$$

```
[In] int(1/(a + b*cos(c + d*x))^(2/3),x)
[Out] int(1/(a + b*cos(c + d*x))^(2/3), x)
```

3.61 $\int \frac{1}{(a+b\cos(c+dx))^{4/3}}\,dx$

Optimal result 390
Rubi [A] (verified) 390
Mathematica [B] (verified) 392
Maple [F] 392
Fricas [F] 392
Sympy [F] 393
Maxima [F] 393
Giac [F] 393
Mupad [F(-1)] 393

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int \frac{1}{(a+b\cos(c+dx))^{4/3}}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}{(a+b)d\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

```
[Out] AppellF1(1/2,4/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*c
os(d*x+c))/(a+b))^(1/3)*sin(d*x+c)*2^(1/2)/(a+b)/d/(a+b*cos(d*x+c))^(1/3)/(
1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2744, 144, 143}

$$\int \frac{1}{(a+b\cos(c+dx))^{4/3}}\,dx = \frac{\sqrt{2}\sin(c+dx)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1}{}\right.}{d(a+b)\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}}$$

```
[In] Int[(a + b*Cos[c + d*x])^(-4/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c +
 d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/((a + b
)*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
```

```
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{\sin(c+dx)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}(a+bx)^{4/3}}\,dx, x, \cos(c+dx)\right)}{d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}$$

$$= -\frac{\left(\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}\sin(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{4/3}}\,dx, x, \cos(c+dx)\right)}{(a+b)d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

$$= \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\sin(c+dx)}{(a+b)d\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 268 vs. $2(110) = 220$.

Time = 2.27 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.44

$$\int \frac{1}{(a+b\cos(c+dx))^{4/3}}\,dx = \frac{15a\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{-\frac{b(1+\cos \cdots}{a \cdots}} \cdots}{\cdots}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(-4/3),x]
```

```
[Out] (15*a*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos
[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 +
 Cos[c + d*x]))/(a - b))]*(a + b*Cos[c + d*x])*Csc[c + d*x] - 6*(5*b^2 + 2*
AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d
*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c +
 d*x]))/(-a + b)]*(a + b*Cos[c + d*x])^2*Csc[c + d*x]^2)*Sin[c + d*x])/(10*
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(1/3))
```

Maple [F]

$$\int \frac{1}{(a+\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

```
[In] int(1/(a+cos(d*x+c)*b)^(4/3),x)
```

```
[Out] int(1/(a+cos(d*x+c)*b)^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(2/3)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{(a+b\cos{(c+dx)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))**(4/3),x)
[Out] Integral((a + b*cos(c + d*x))**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

```
[In] int(1/(a + b*cos(c + d*x))^(4/3),x)
[Out] int(1/(a + b*cos(c + d*x))^(4/3), x)
```

3.62 $\int (a+b\cos(c+dx))^n\,dx$

Optimal result 394
Rubi [A] (verified) 394
Mathematica [A] (verified) 396
Maple [F] 396
Fricas [F] 396
Sympy [F] 396
Maxima [F] 397
Giac [F] 397
Mupad [F(-1)] 397

Optimal result

Integrand size = 12, antiderivative size = 103

$$\int (a+b\cos(c+dx))^n\,dx$$
$$=\frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^n\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{-n}\sin(c+d}{d\sqrt{1+\cos(c+dx)}}$$

```
[Out] AppellF1(1/2,-n,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos
(d*x+c))^n*sin(d*x+c)*2^(1/2)/d/(((a+b*cos(d*x+c))/(a+b))^n)/(1+cos(d*x+c))
^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2744, 144, 143}

$$\int (a+b\cos(c+dx))^n\,dx$$
$$=\frac{\sqrt{2}\sin(c+dx)(a+b\cos(c+dx))^n\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx)}{a+b}\right.}{d\sqrt{\cos(c+dx)+1}}$$

```
[In] Int[(a + b*Cos[c + d*x])^n,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -n, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c +
d*x]))/(a + b)]*(a + b*Cos[c + d*x])^n*Sin[c + d*x])/(d*Sqrt[1 + Cos[c + d*
x]]*((a + b*Cos[c + d*x])/(a + b))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{\sin(c+dx)\text{Subst}\left(\int \frac{(a+bx)^n}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}$$

$$= -\frac{\left((a+b\cos(c+dx))^n\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{-n}\sin(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^n}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+d$$

$$d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}$$

$$= \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^n\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{-n}}{d\sqrt{1+\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.17

$$\int (a+b\cos(c+dx))^n\,dx = -\frac{\text{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}(a+b\cos(c+dx}{bd(1+n)}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^n,x]
[Out] -((AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Cos[c + d*x])/(a - b), (a + b*Co
s[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 +
Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])^(1 + n)*Csc[c + d*x])/(b*d*(1
 + n)))
```

Maple [F]

$$\int (a+\cos(dx+c)\,b)^n\,dx$$

```
[In] int((a+cos(d*x+c)*b)^n,x)
[Out] int((a+cos(d*x+c)*b)^n,x)
```

Fricas [F]

$$\int (a+b\cos(c+dx))^n\,dx = \int (b\cos(dx+c)+a)^n\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c) + a)^n, x)
```

Sympy [F]

$$\int (a+b\cos(c+dx))^n\,dx = \int (a+b\cos(c+dx))^n\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**n,x)
[Out] Integral((a + b*cos(c + d*x))**n, x)
```

Maxima [F]

$$\int (a+b\cos(c+dx))^n\,dx = \int (b\cos(dx+c)+a)^n\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^n\,dx = \int (b\cos(dx+c)+a)^n\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^n\,dx = \int (a+b\,\cos(c+d\,x))^n\,dx$$

```
[In] int((a + b*cos(c + d*x))^n,x)
[Out] int((a + b*cos(c + d*x))^n, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 399

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                      convert(ExpnType_result,string)," vs. order ",
                      convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

  #debug=False
  if debug:
     print ("Enter is_atom, expn=",expn)

  if not hasattr(expn, 'parent'):
    return False


  #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
  try:
    if expn.parent() is SR:
      return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
      return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
      return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

  except AttributeError as error:
    print("Exception,AttributeError in  is_atom")
    print ("cought exception" , type(error).__name__ )
    return False


def expnType(expn):

  if debug:
    print (">>>>>Enter expnType, expn=", expn)
    print (">>>>>is_atom(expn)=", is_atom(expn))

  if is_atom(expn):
    return 1
  elif type(expn)==list:   #isinstance(expn,list):
    return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
      return 1
    else:
      return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
  elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
    if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
      return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
      if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```python
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```